Physics and Results of CASCADE.04 model

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Physics Models

Intra-nuclear cascade model

Pre-equilibrium (exciton model)

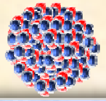
Evaporation (Generalized Evaporation Model)

Fission model (Fong's Model)

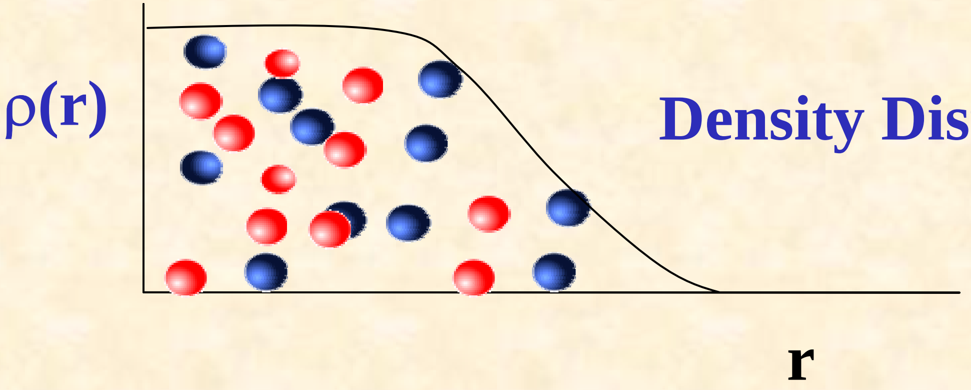
Intra-Nuclear Cascade model

What are the inputs we have?

Projectile:  **Charge, mass, energy/momentum**

Target:  **Charge, mass, nucleon density distribution**

Each nucleon is assigned position & momentum



Density Distribution

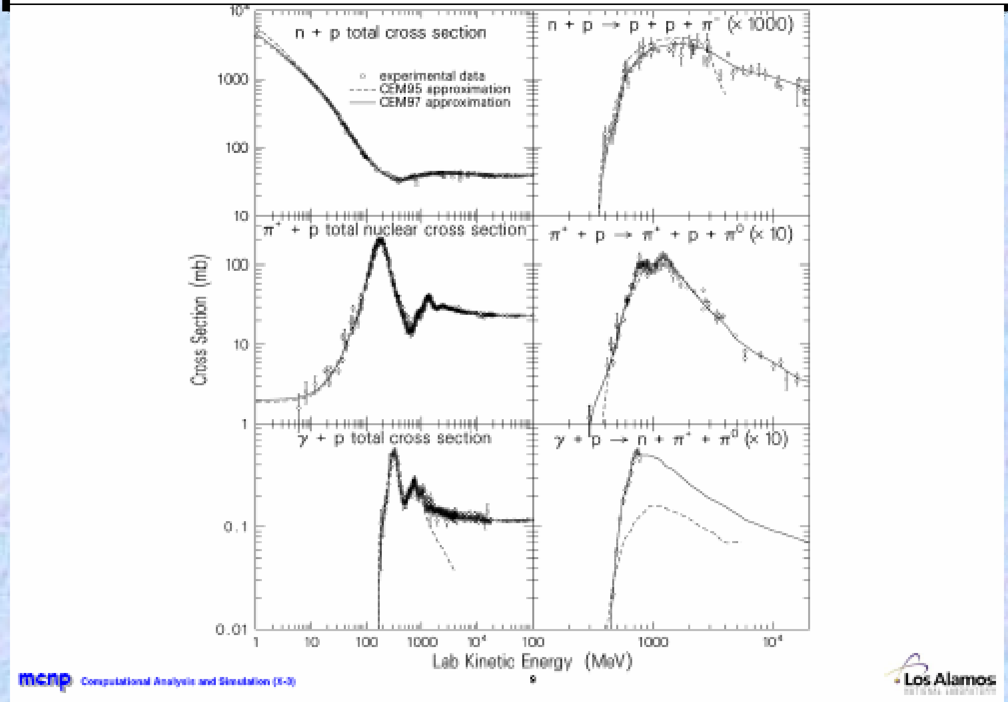
$$\left\{\begin{array}{ll} \rho(r) = \frac{\rho_0}{1+exp\frac{(r-r_0)}{a}} & \\ \text{where} & r_0 = 1.07A^{1/3}fm \\ a = 0.545fm & \text{For A} > 10 \\ \rho(r) = \rho_0 exp\frac{-r^2}{R^2} & \text{For A} \leq 10 \end{array}\right\}$$

$$\left\{\begin{array}{l} P_F(r) = (\frac{3\pi^2\rho(r)}{2})^{1/3} \\ E_F(r) = \hbar^2\frac{(3\pi^2\rho(r))^{2/3}}{2m_N} \end{array}\right\}$$

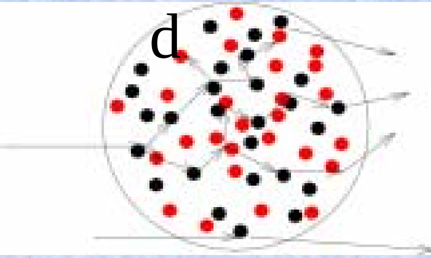
$$\left\{\begin{array}{l} V \equiv V_N = E_F + \text{Bindinging energy} \\ V_\pi = 25\text{MeV} \end{array}\right\}$$

Intra-Nuclear Cascade model

$$NN \to NN, \qquad NN \to \pi NN, \qquad NN \to \pi_1, \cdots, \pi_i NN$$
$$\pi N \to \pi N, \qquad \pi N \to \pi_1, \cdots, \pi_i N \qquad (i \geq 2)$$



$\lambda = p/m$



$$\boxed{\lambdabar \ll d}$$

$$\boxed{\lambdabar \ll \Lambda}$$

Reactions

$$\left\{\begin{array}{ll} p+p=p+p & \text{Isotropic E} < 0.46 \text{ GeV} \\ p+p=p+p & 0.46 < \text{E} < 2.8 \text{ GeV} \\ p+p=p+p & 2.8 < \text{E} < 10.0 \text{ GeV} \\ p+n=p+n & \text{E} < 0.97 \text{ GeV} \\ \pi^+ + p = \pi^+ + p & \text{E} < 80.0 \text{ MeV} \\ \pi^+ + p = \pi^+ + p & 80 < \text{E} < 300.0 \text{ MeV} \\ \pi^+ + p = \pi^+ + p & 0.3 < \text{E} < 1.0 \text{ GeV} \\ \pi^+ + p = \pi^+ + p & 1.0 < \text{E} < 2.4 \text{ GeV} \end{array}\right.$$

Cross-section

$$\left\{\begin{array}{l} cos(\theta) = 2\xi^{1/2}[\sum\limits_{n=0}^{N} a_n \xi^n + (1 - \sum\limits_{n=0}^{N} a_n)\xi^{N+1}] - 1 \\ a_n = \sum\limits_{k=0}^{N} a_{nk} E^k \\ \text{N=3, M=3} \end{array}\right\}$$

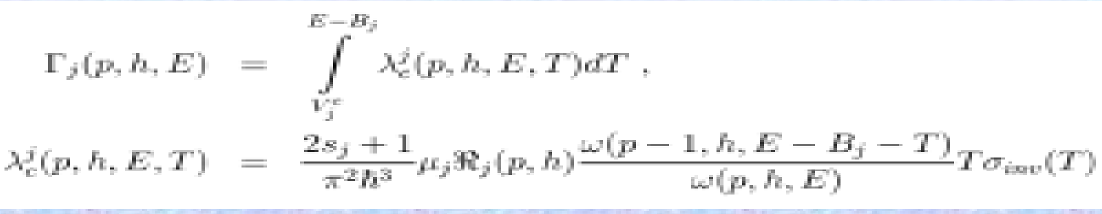
Angular distribution

Pre-equilibrium model (Exciton model)

Sharp cut off energy (7 MeV) is the criteria to close INC

n, p, d, t, ^{3}He, and ^{4}He emission

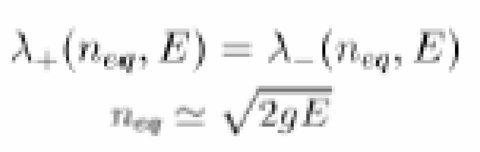
Probability of emission is calculated as given below



$$\Gamma_j(p,h,E) = \int_{V_j^c}^{E-B_j} \lambda_c^j(p,h,E,T)\,dT\ ,$$

$$\lambda_c^j(p,h,E,T) = \frac{2s_j+1}{\pi^2\hbar^3}\mu_j \Re_j(p,h)\frac{\omega(p-1,h,E-B_j-T)}{\omega(p,h,E)} T\sigma_{inv}(T)$$

p=particle, h=hole, n=p+h is exciton number, s=spin, $\sigma_{\rm inv}$=cross-section, E=excitation energy, B=binding energy

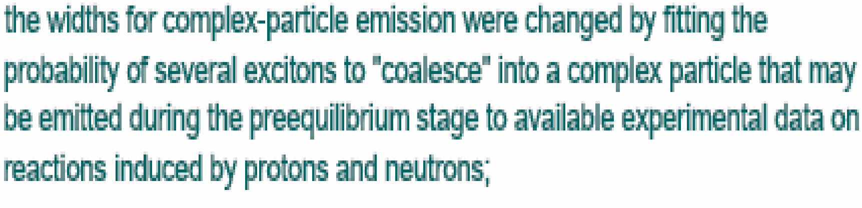


$$\lambda_+(n_{eq},E) = \lambda_-(n_{eq},E)$$
$$n_{eq} \simeq \sqrt{2gE}$$

M. Veselsky, Nucl. Phys. A705 (2002) 193

Differences with CEM

Kalbach Systematics below 210 MeV



the widths for complex-particle emission were changed by fitting the probability of several excitons to "coalesce" into a complex particle that may be emitted during the preequilibrium stage to available experimental data on reactions induced by protons and neutrons;

Earlier assumption

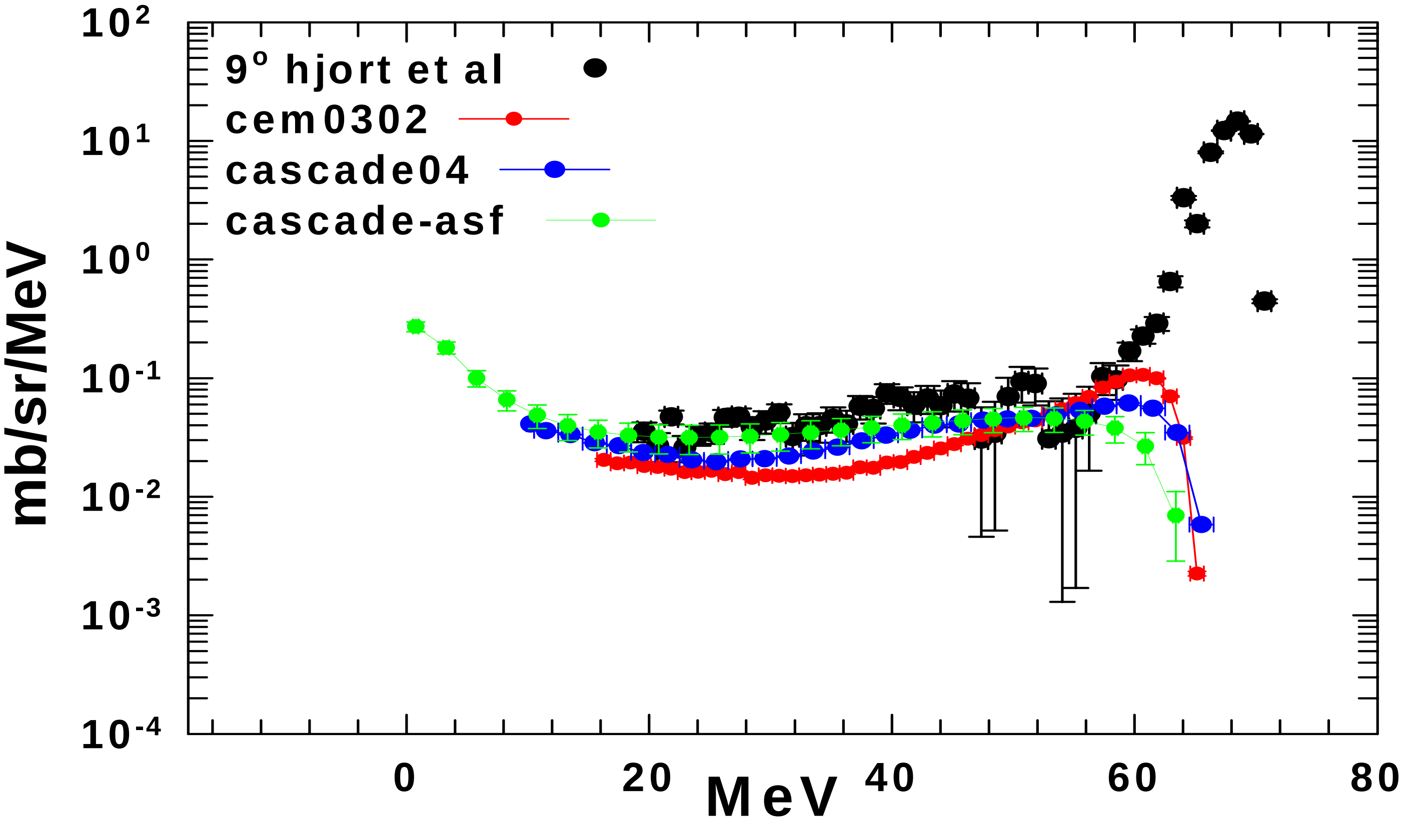
P_{pre}=1 or 0

Now

$$P_{pre}(n/n_{eq}) = 1 - \exp\left(-\frac{(n/n_{eq}-1)^2}{2\sigma_{pre}^2}\right)$$

$\sigma_{\rm pre}$=0.22 (0.4 in CEM)

P (65MeV)+^{56}Fe



mb/sr/MeV
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
9° hjort et al
cem0302
cascade04
cascade-asf
0
20
40
60
80
MeV

Evaporation model

$$P_j(\epsilon)d\epsilon = g_j\sigma_{inv}(\epsilon)\frac{\rho_d(E-Q-\epsilon)}{\rho_i(E)}\epsilon d\epsilon$$

$$P_f = \frac{\int_0^{U-B_f}\rho(U-B_f-\delta-E)dE}{\rho_j(E)}$$

$$\rho(E) = \frac{c_1 exp(2\sqrt{a(E-\delta)})}{a^{1/4}(E-\delta)^{5/4}}$$

$$\rho(E) = c_2 exp((E-E_0)/T)$$

Z_j	Ejectiles						
0	n						
1	p	d	t				
2	^{3}He	^{4}He	^{6}He	^{8}He			
3	^{6}Li	^{7}Li	^{8}Li	^{9}Li			
4	^{7}Be	^{9}Be	^{10}Be	^{11}Be	^{12}Be		
5	^{8}B	^{10}B	^{11}B	^{12}B	^{13}B		
6	^{10}C	^{11}C	^{12}C	^{13}C	^{14}C	^{15}C	^{16}C
7	^{12}N	^{13}N	^{14}N	^{15}N	^{16}N	^{17}N	
8	^{14}O	^{15}O	^{16}O	^{17}O	^{18}O	^{19}O	^{20}O
9	^{17}F	^{18}F	^{19}F	^{20}F	^{21}F		
10	^{18}Ne	^{19}Ne	^{20}Ne	^{21}Ne	^{22}Ne	^{23}Ne	^{24}Ne
11	^{21}Na	^{22}Na	^{23}Na	^{24}Na	^{25}Na		
12	^{22}Mg	^{23}Mg	^{24}Mg	^{25}Mg	^{26}Mg	^{27}Mg	^{28}Mg

S is assumed 0 at saddle

$$a(A_d, Z_d, E) = A_d(0.134 - 1.2110^{-04}A_d)(1 + \frac{S}{E}(1 - exp(-0.061E)))$$

$$a_f = a_n\left\{\begin{array}{ll} 1.041 + 0.00915X^2 - 0.0005977X^3 & \text{For Z} < 78 \\ 1.0196 + 0.00896X^2 - 0.000585X^3 & \text{For } 78 < \text{Z} < 85 \\ 0.9445 + 0.0083X^2 - 0.000542X^3 & \text{For Z} > 85 \end{array}\right\}$$

Fission barrier from Myers and Swiatecki systematics

Fission model

$$\Omega(E_1) = c_1 exp(2\sqrt{a_1 E_1})$$
$$\Omega(E_2) = c_2 exp(2\sqrt{a_2 E_2})$$

$$E_1 : E_2 = a_1 T^2 : a_2 T^2 = a_1 : a_2$$

scission point model

$$\Omega(E) = c_1 c_2 \int_0^E exp(2\sqrt{(a_1 + a_2)(E_1 + E_2)})$$

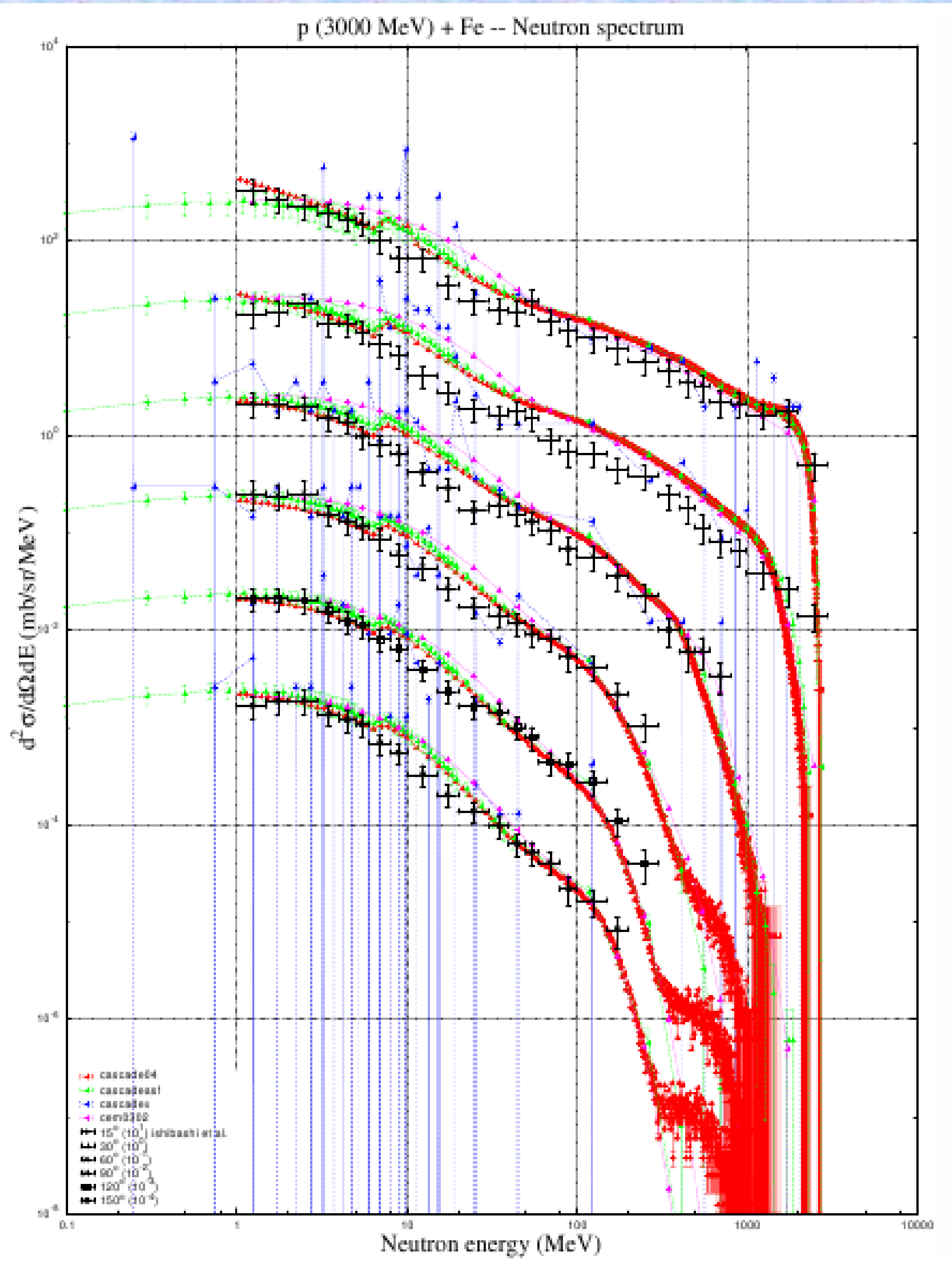
E = M*(A, Z)-M(A1, Z1)-M(A2, Z2) - coulomb energy – deformation energy

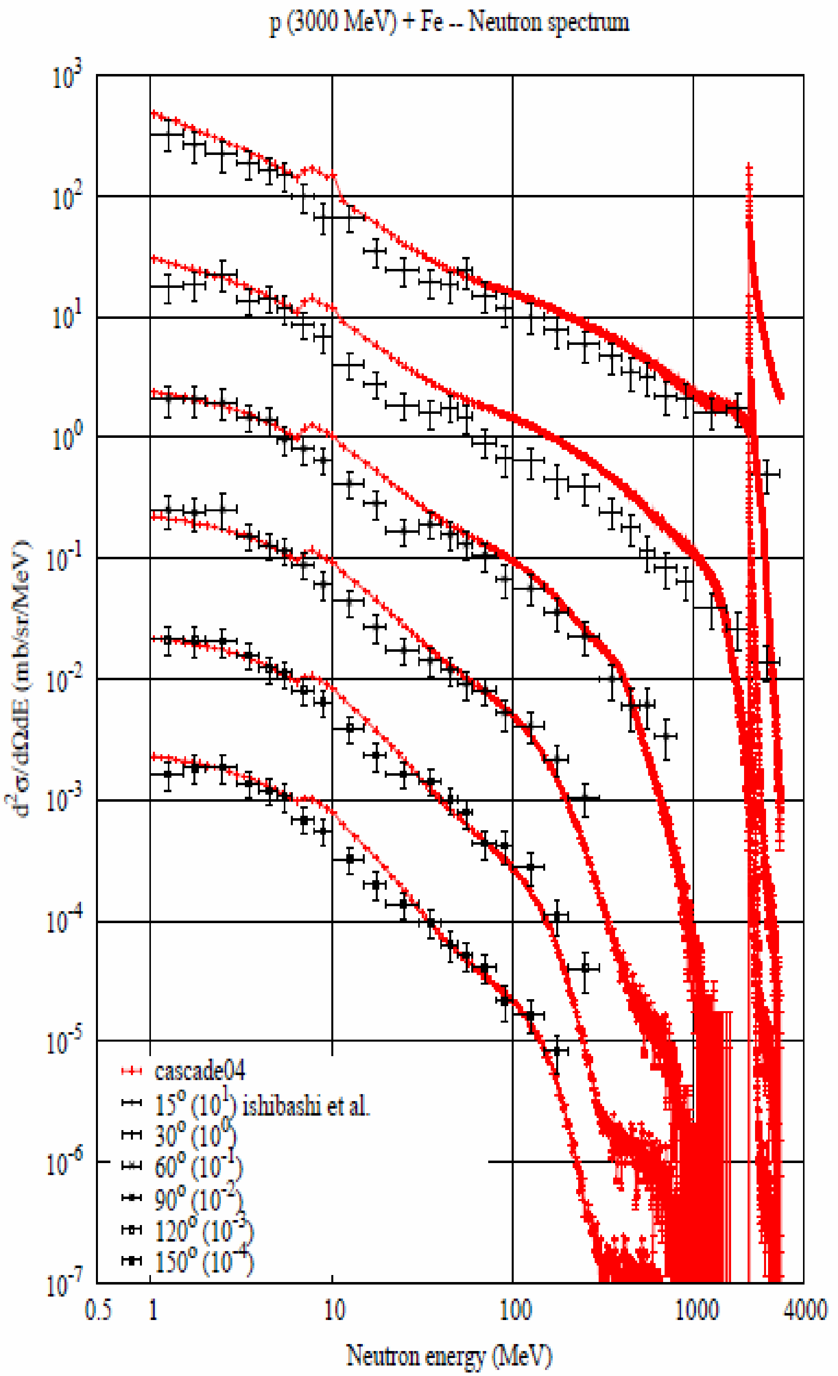
$$E_1 : E_2 = a_1 T^2 : a_2 T^2 = a_1 : a_2$$

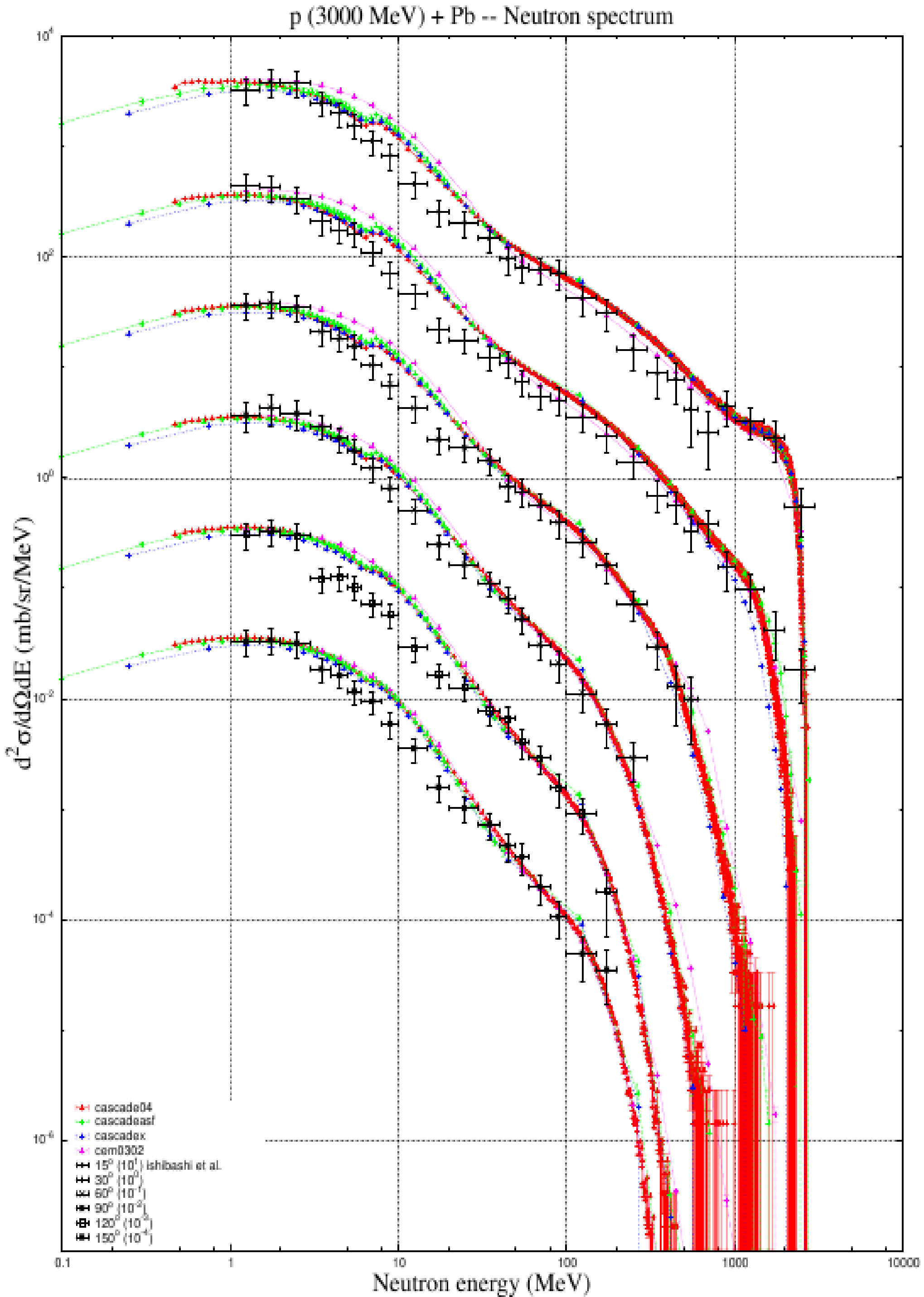
$$V_C = \frac{1.44 Z_1 Z_2}{R_{12}}$$
$$R_{12} = R_0(1 + \alpha_{2i}(1 - \tfrac{3}{5}X_i) + \alpha_{3i}(1 - \tfrac{3}{7}X_i^2))$$
$$X_i = \frac{R_0(i)}{\sum_i R_0(i)(1+\alpha_{2i}+\alpha_{3i}-\frac{9}{35}\alpha_{2i}\alpha_{3i})}$$
$$R_0(i) = R_0 A_i^{1/3}$$

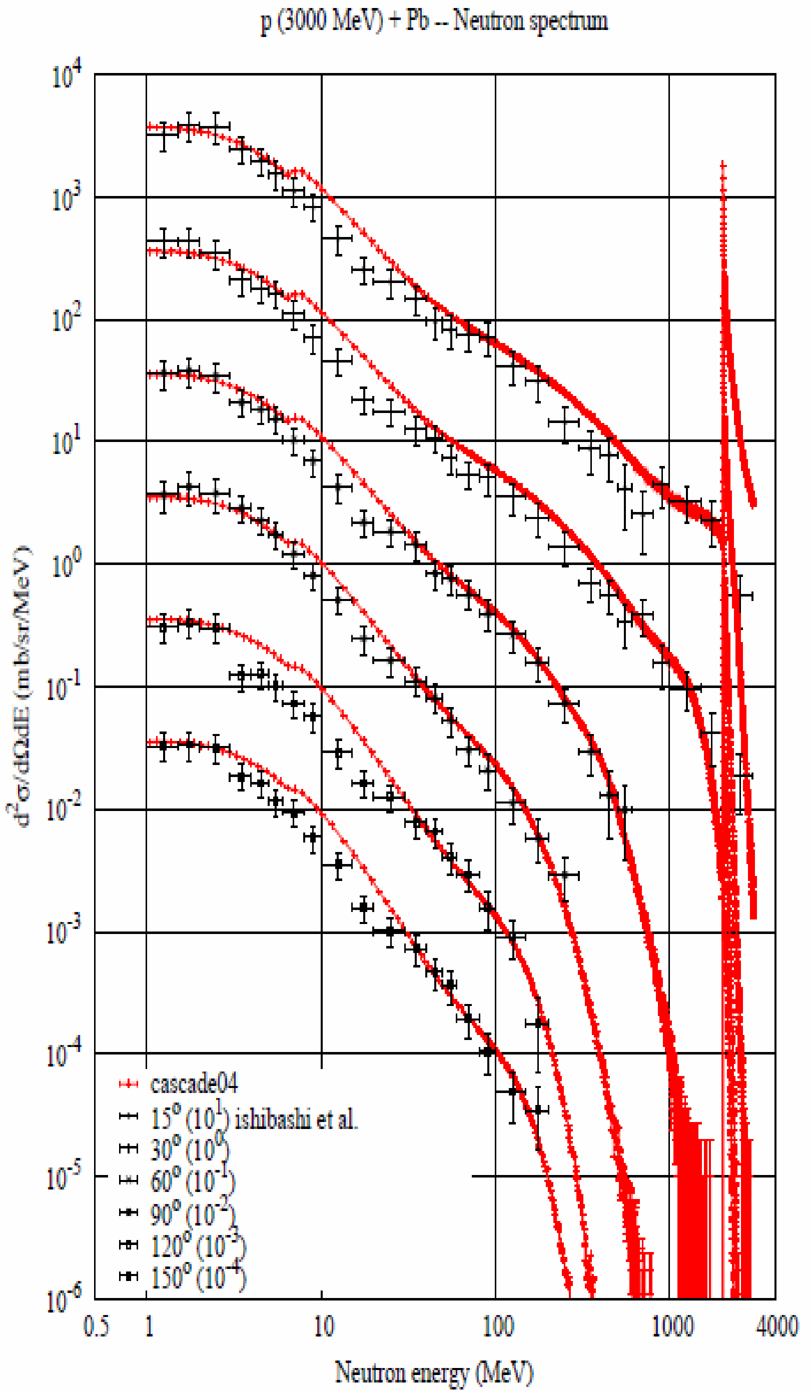
and $\mathrm{R_0} = 1.3$

$$D(i) = (0.4E_S - 0.2E_C)\alpha_{2i}^2 + (0.7144E_S - 0.204E_C)\alpha_{3i}^2$$

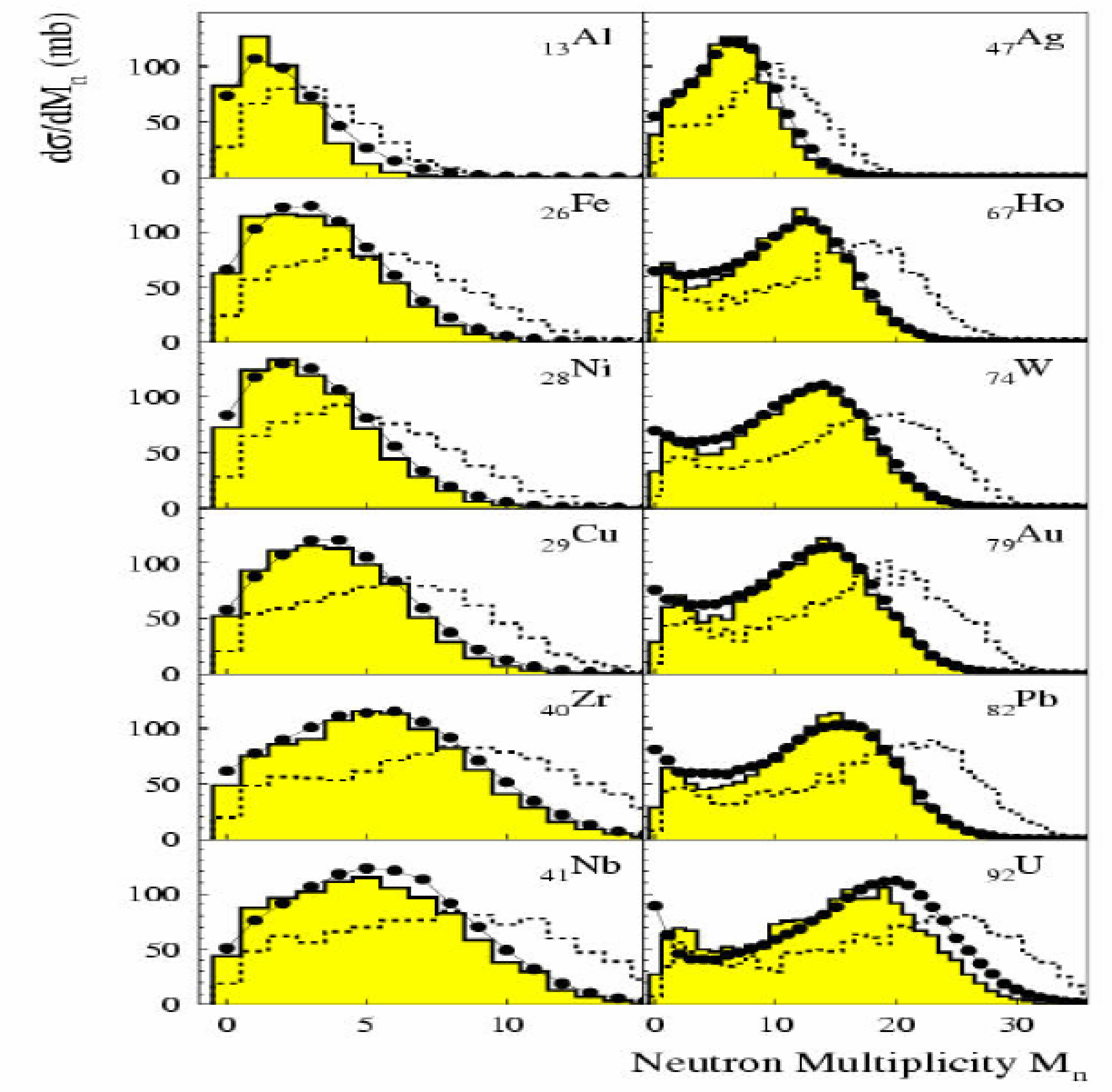

p (3000 MeV) + Fe -- Neutron spectrum
$d^2\sigma/d\Omega dE$ (mb/sr/MeV)
Neutron energy (MeV)
10^4
10^2
10^0
10^-2
10^-4
10^-6
10^-8
0.1
1
10
100
1000
10000
cascade04
cascadeasl
cascadex
cem0302
15° (10^1) ishibashi et al.
30° (10^0)
60° (10^-1)
90° (10^-2)
120° (10^-3)
150° (10^-4)

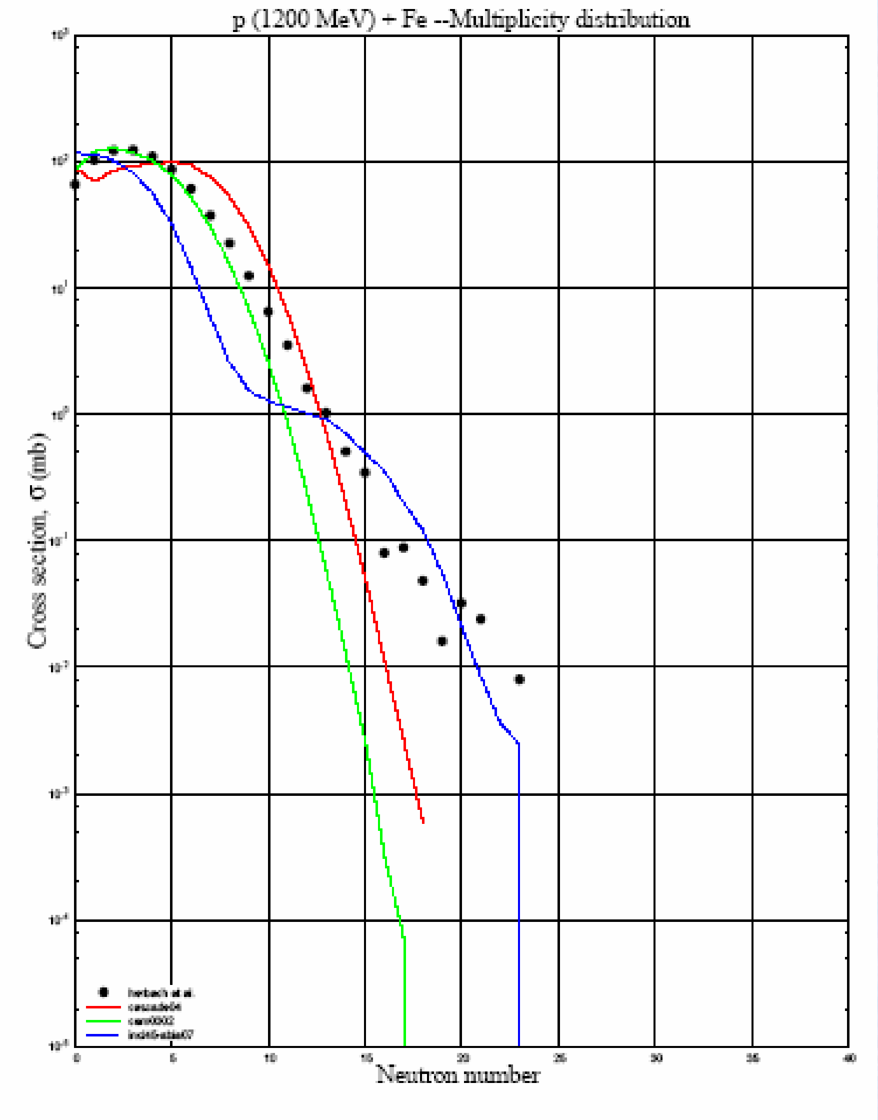

p (3000 MeV) + Fe -- Neutron spectrum
$d^2\sigma/d\Omega dE$ (mb/sr/MeV)
Neutron energy (MeV)
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
0.5
1
10
100
1000
4000
cascade04
15° (10^1) ishibashi et al.
30° (10^0)
60° (10^-1)
90° (10^-2)
120° (10^-3)
150° (10^-4)


p (3000 MeV) + Pb -- Neutron spectrum
d^{2}σ/dΩdE (mb/sr/MeV)
Neutron energy (MeV)
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
0.1
1
10
100
1000
10000
cascade04
cascadeasf
cascadex
cem0302
15° (10^1) ishibashi et al.
30° (10^0)
60° (10^{-1})
90° (10^{-2})
120° (10^{-3})
150° (10^{-4})

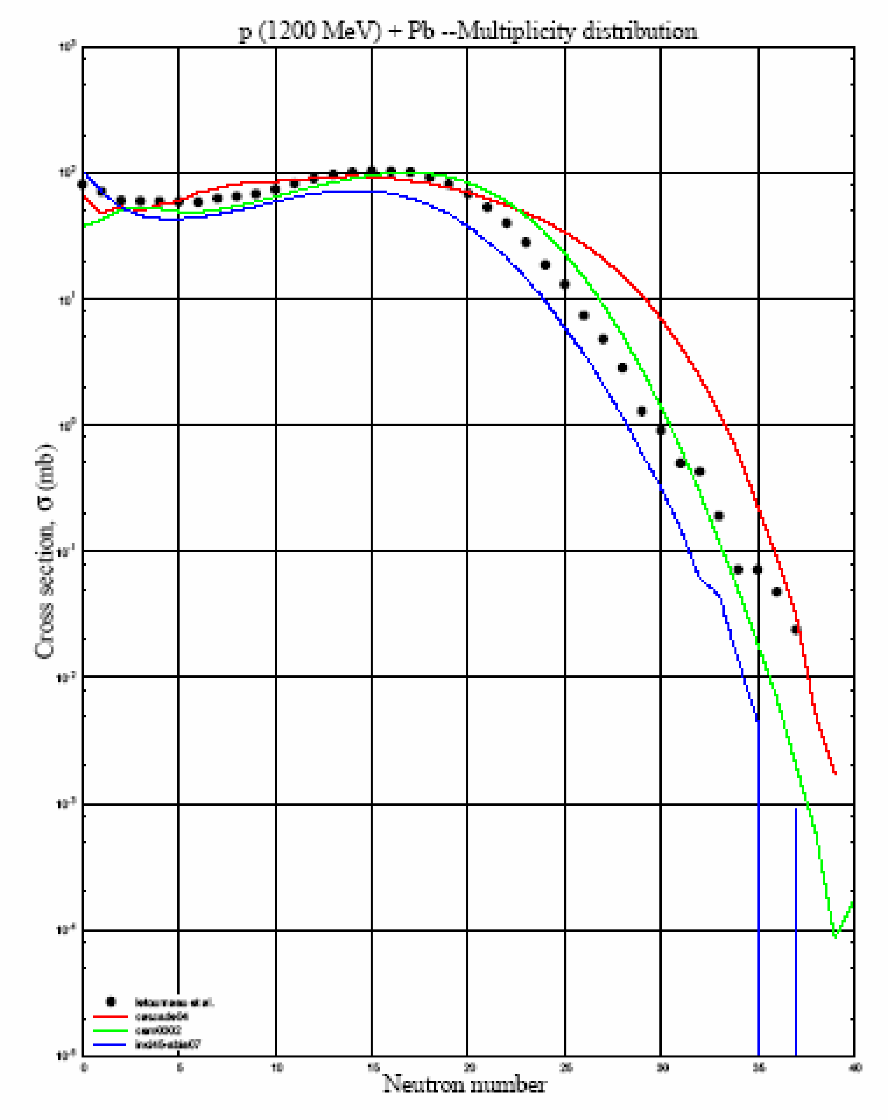

p (3000 MeV) + Pb -- Neutron spectrum
d^{2}σ/dΩdE (mb/sr/MeV)
Neutron energy (MeV)
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.5
1
10
100
1000
4000
cascade04
15° (10^1) ishibashi et al.
30° (10^0)
60° (10^{-1})
90° (10^{-2})
120° (10^{-3})
150° (10^{-4})

The multiplicity data impose strong boundary on full code but still not on individual models

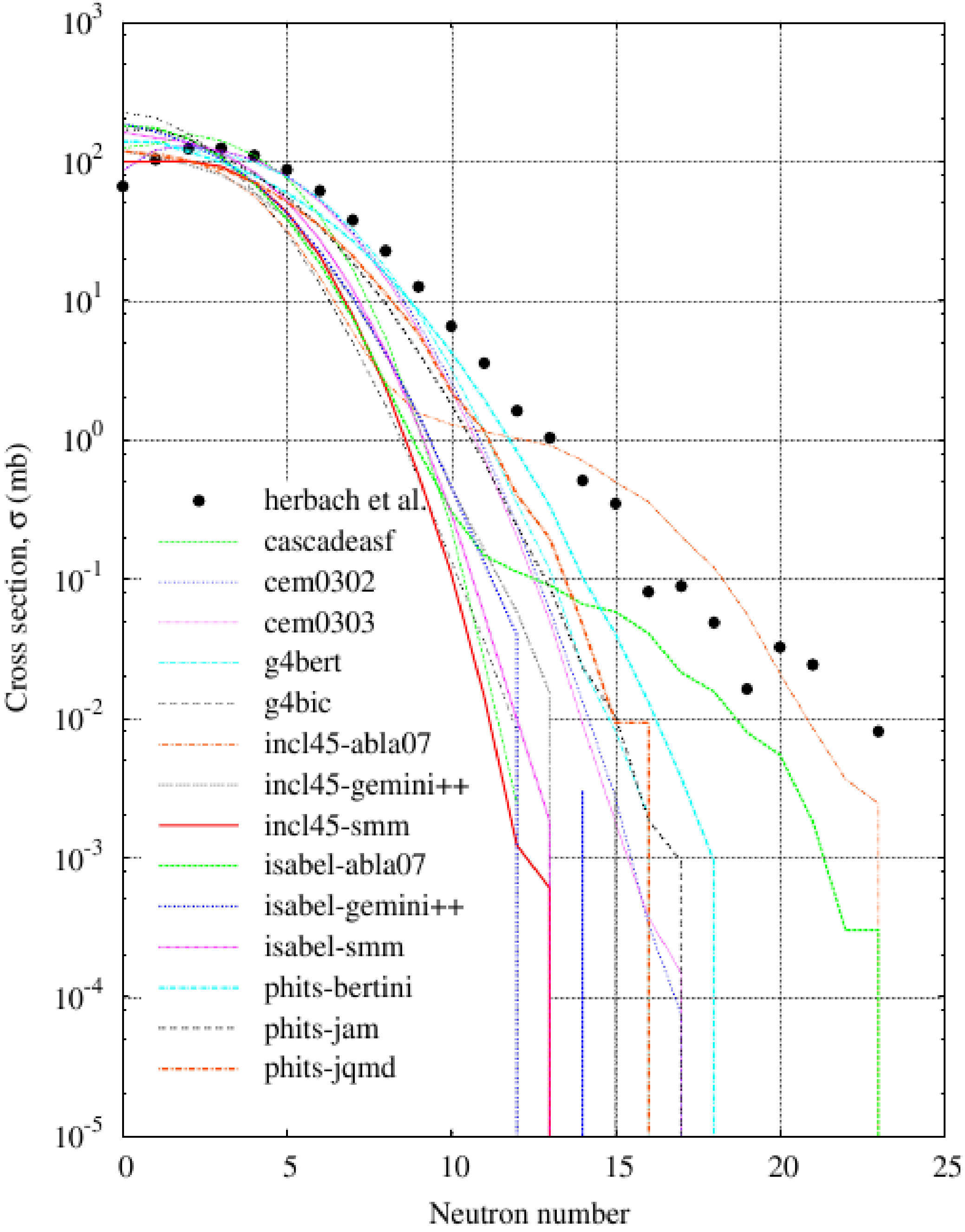


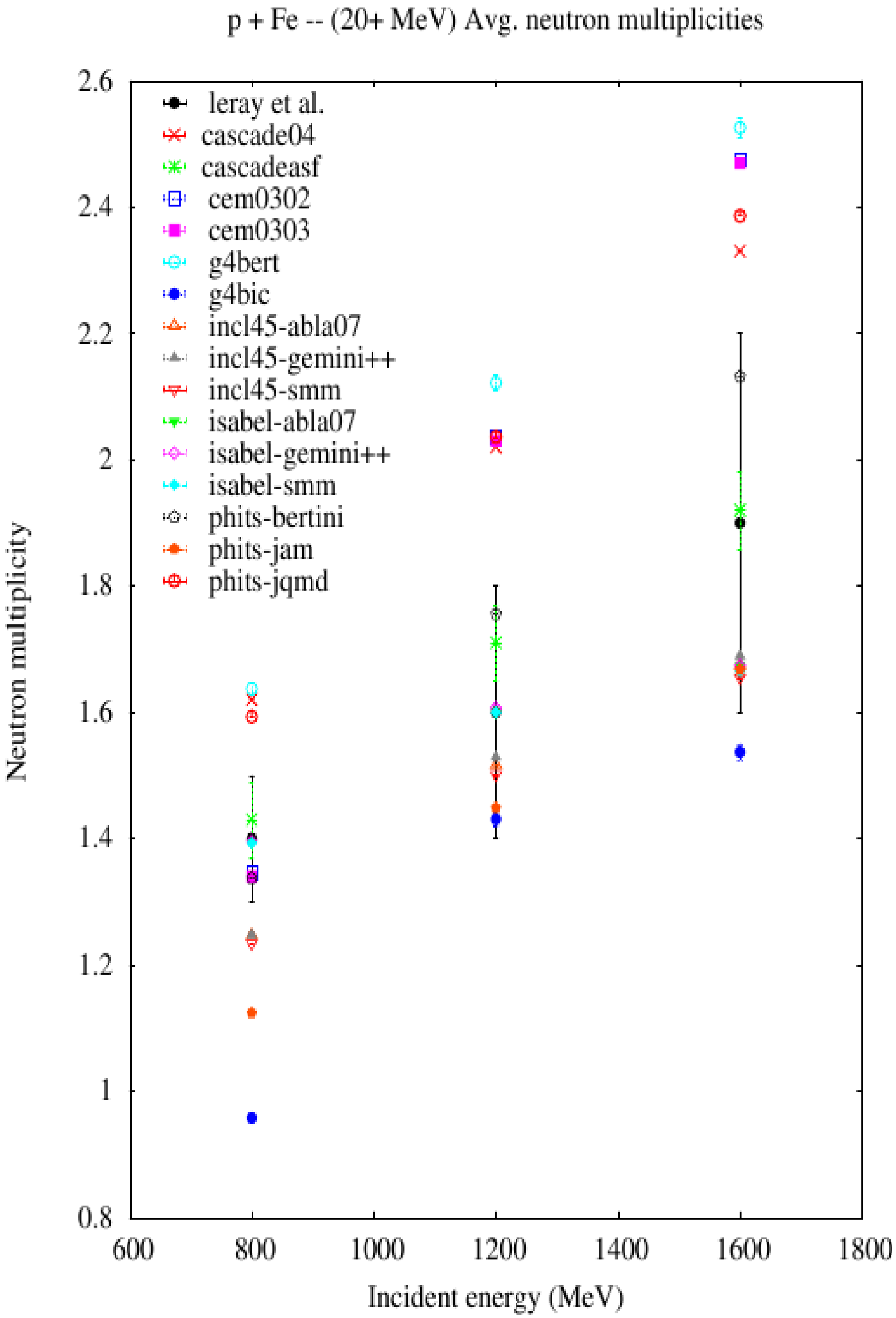


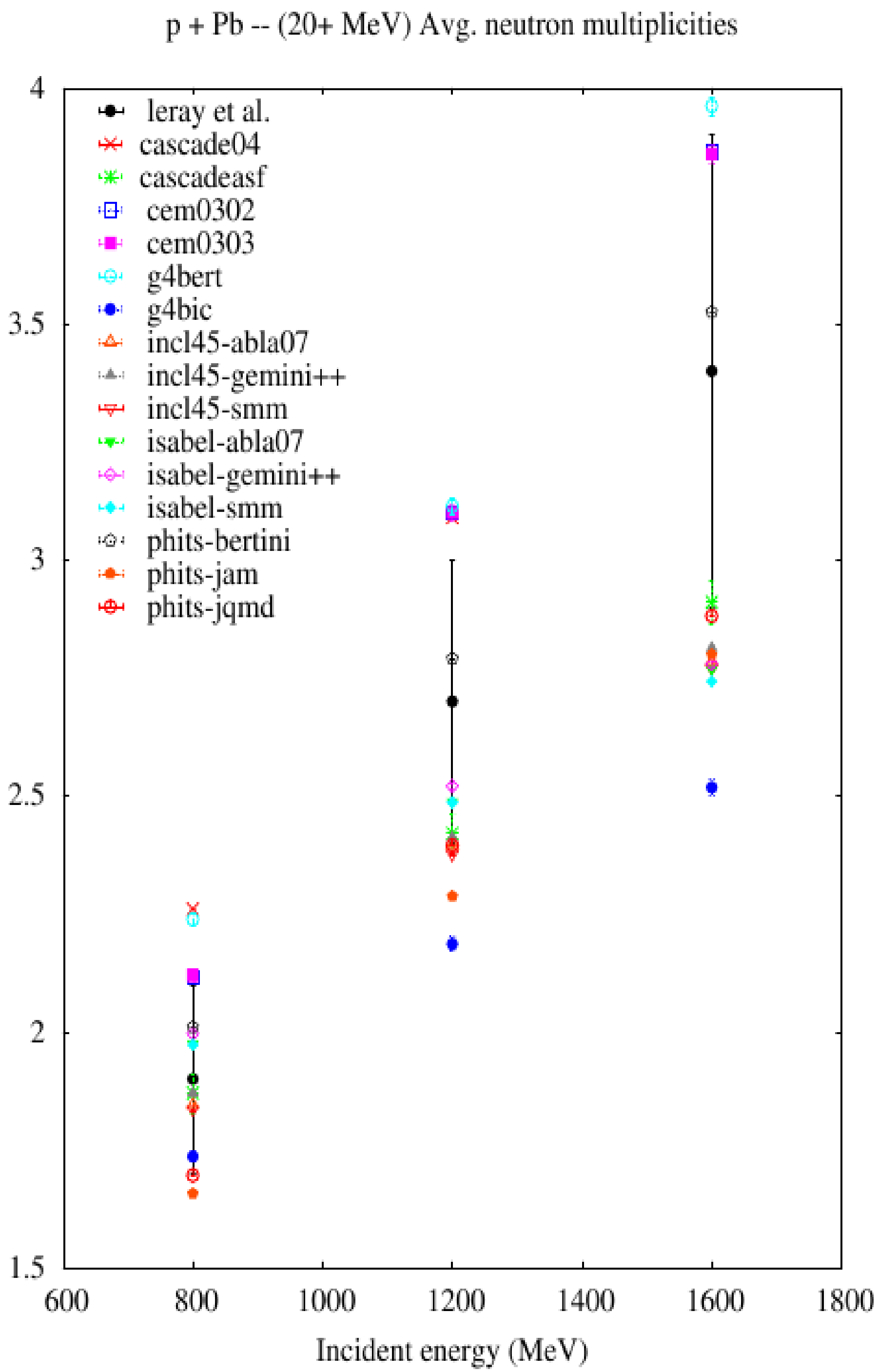
p (1200 MeV) + Fe --Multiplicity distribution

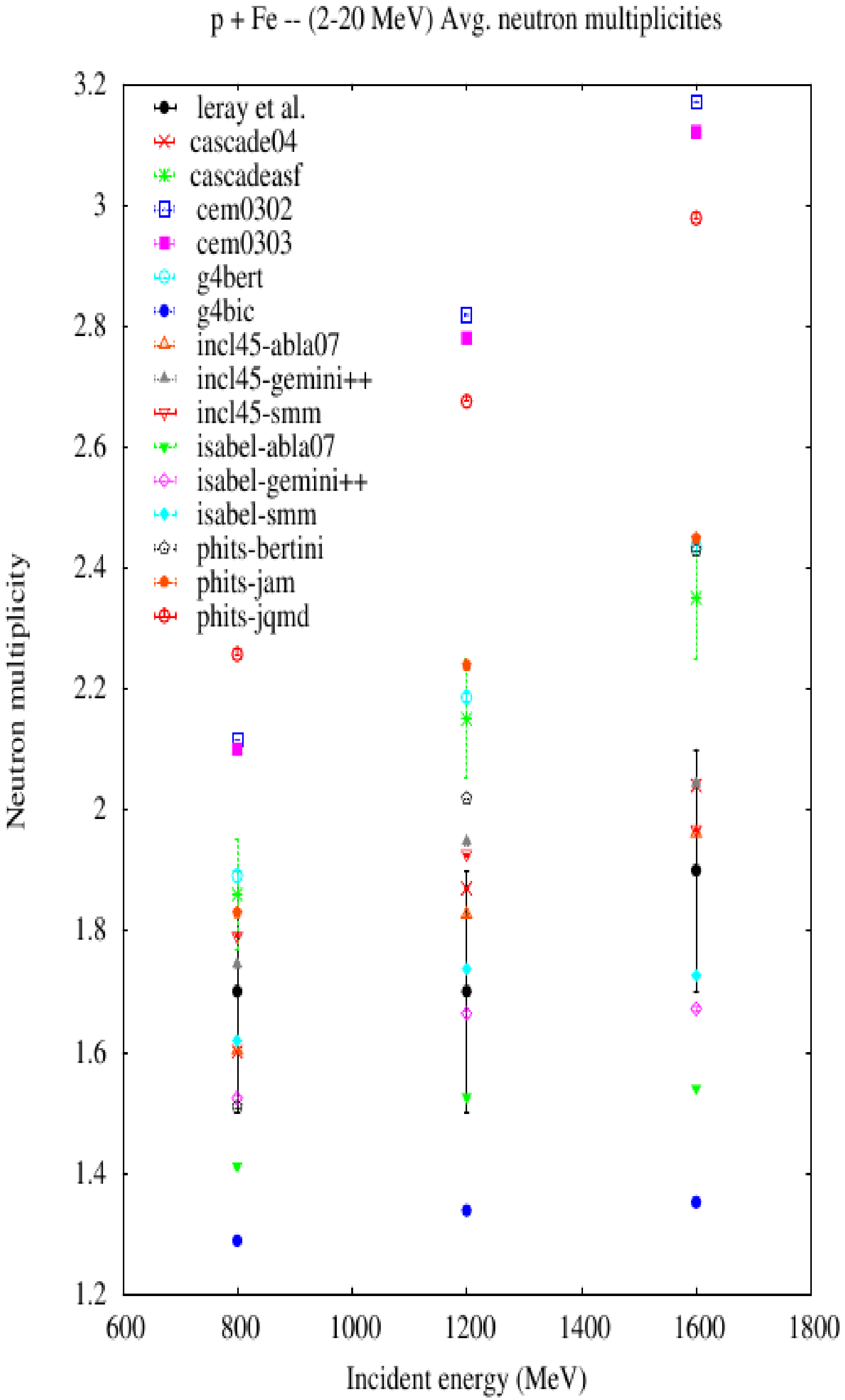


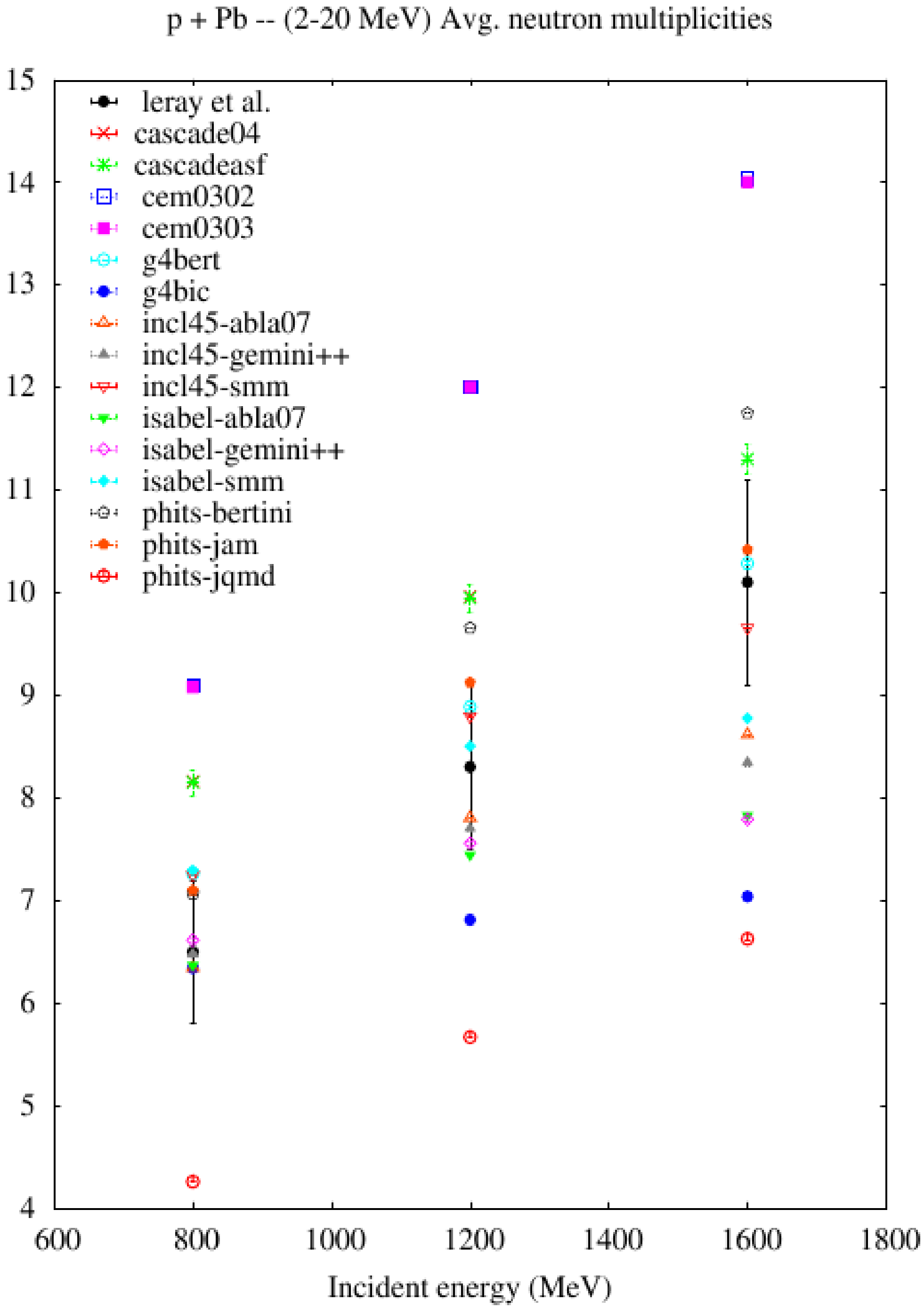
p (1200 MeV) + Pb --Multiplicity distribution p (1200 MeV) + Fe --Multiplicity distribution




p + Fe -- (20+ MeV) Avg. neutron multiplicities
leray et al.
cascade04
cascadeasf
cem0302
cem0303
g4bert
g4bic
incl45-abla07
incl45-gemini++
incl45-smm
isabel-abla07
isabel-gemini++
isabel-smm
phits-bertini
phits-jam
phits-jqmd
Neutron multiplicity
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
600
800
1000
1200
1400
1600
1800
Incident energy (MeV)


p + Pb -- (20+ MeV) Avg. neutron multiplicities
leray et al.
cascade04
cascadeasf
cem0302
cem0303
g4bert
g4bic
incl45-abla07
incl45-gemini++
incl45-smm
isabel-abla07
isabel-gemini++
isabel-smm
phits-bertini
phits-jam
phits-jqmd
1.5
2
2.5
3
3.5
4
600
800
1000
1200
1400
1600
1800
Incident energy (MeV)


p + Fe -- (2-20 MeV) Avg. neutron multiplicities
Neutron multiplicity
Incident energy (MeV)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
600
800
1000
1200
1400
1600
1800
leray et al.
cascade04
cascadeasf
cem0302
cem0303
g4bert
g4bic
incl45-abla07
incl45-gemini++
incl45-smm
isabel-abla07
isabel-gemini++
isabel-smm
phits-bertini
phits-jam
phits-jqmd


p + Pb -- (2-20 MeV) Avg. neutron multiplicities
Incident energy (MeV)
15
14
13
12
11
10
9
8
7
6
5
4
600
800
1000
1200
1400
1600
1800
leray et al.
cascade04
cascadeasf
cem0302
cem0303
g4bert
g4bic
incl45-abla07
incl45-gemini++
incl45-smm
isabel-abla07
isabel-gemini++
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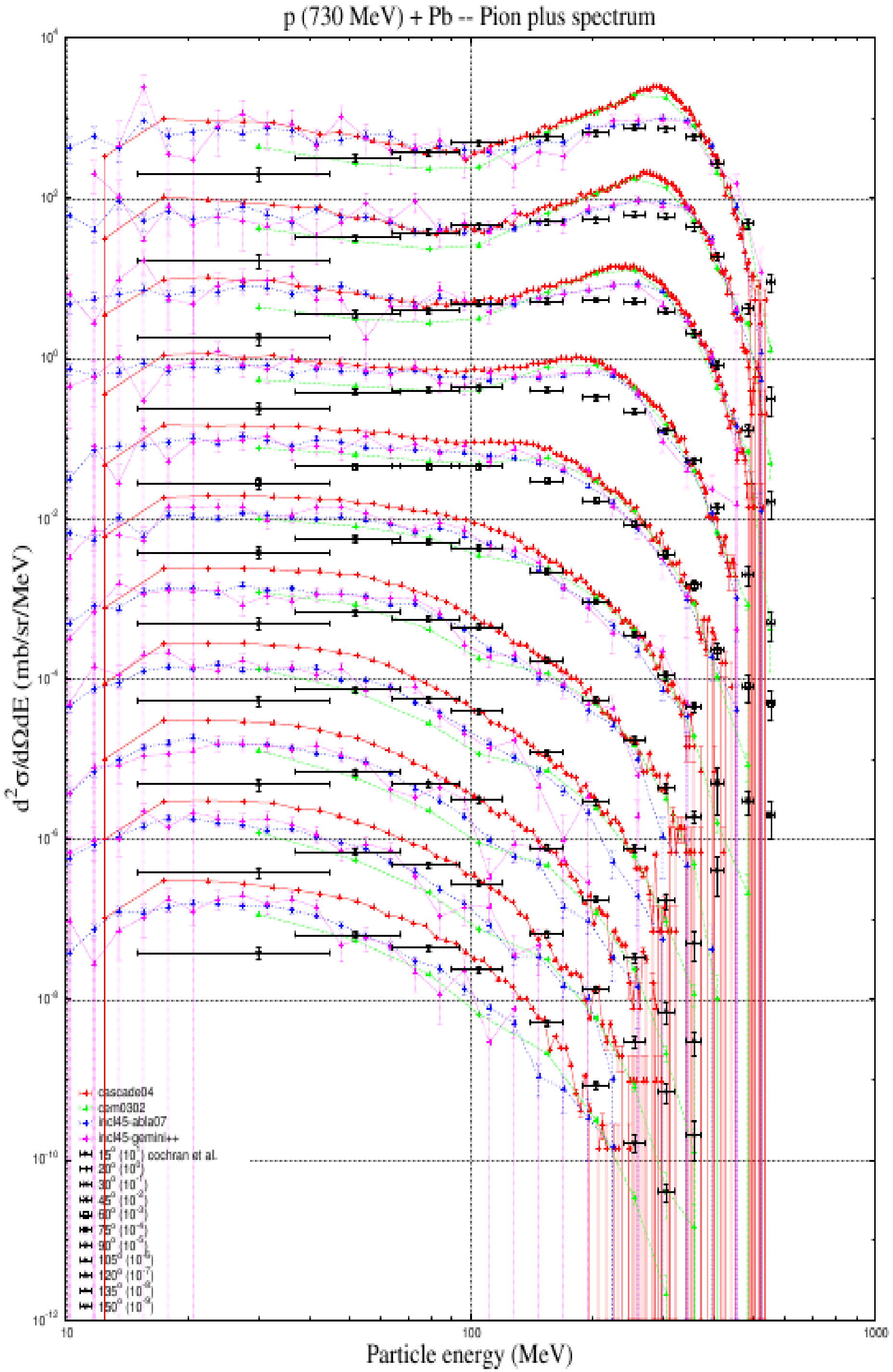
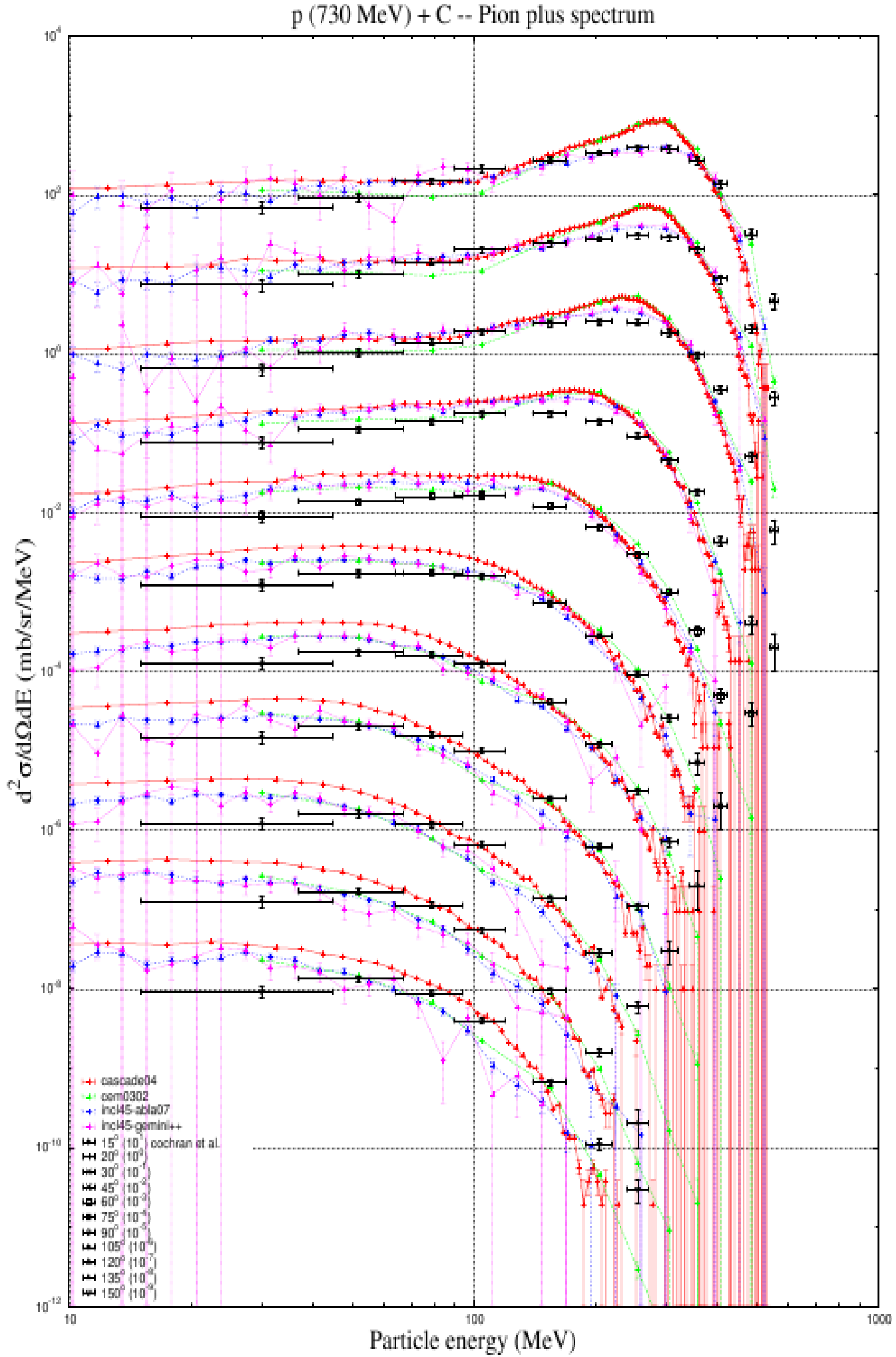
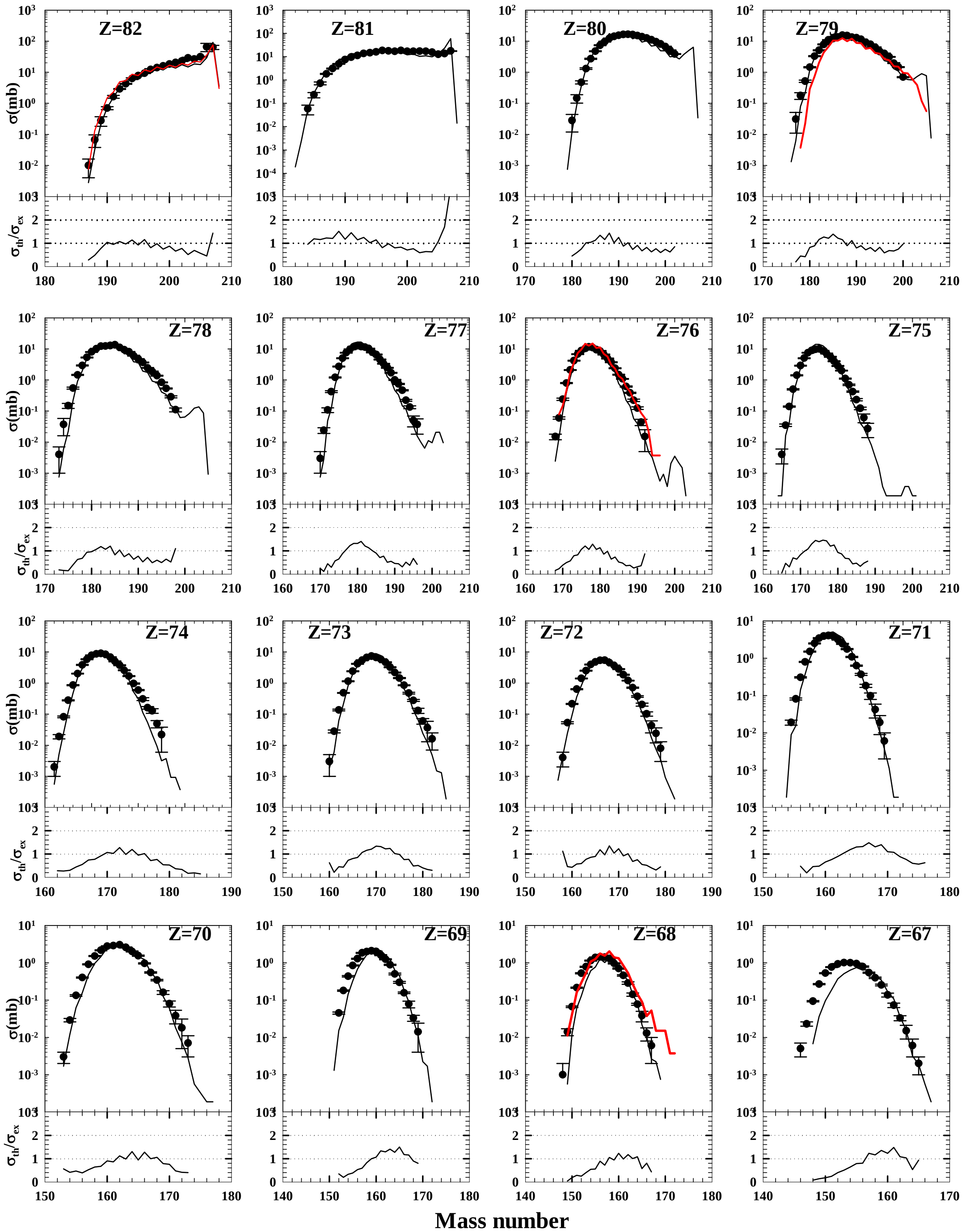
phits-jqmd


p (730 MeV) + Pb -- Pion plus spectrum
$d^2\sigma/d\Omega dE$ (mb/sr/MeV)
Particle energy (MeV)
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
100
1000
cascade04
cem0302
incl45-abla07
incl45-gemini++
15° (10^1) cochran et al.
20° (10^0)
30° (10^{-1})
45° (10^{-2})
60° (10^{-3})
75° (10^{-4})
90° (10^{-5})
105° (10^{-6})
120° (10^{-7})
135° (10^{-8})
150° (10^{-9})


p (730 MeV) + C -- Pion plus spectrum
$d^2\sigma/d\Omega dE$ (mb/sr/MeV)
Particle energy (MeV)
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
100
1000
cascade04
cem0302
incl45-abla07
incl45-gemini++
15° (10^1) cochran et al.
20° (10^0)
30° (10^{-1})
45° (10^{-2})
60° (10^{-3})
75° (10^{-4})
90° (10^{-5})
105° (10^{-6})
120° (10^{-7})
135° (10^{-8})
150° (10^{-9})



P (1GeV)+^{208}Pb

Earlier assumption

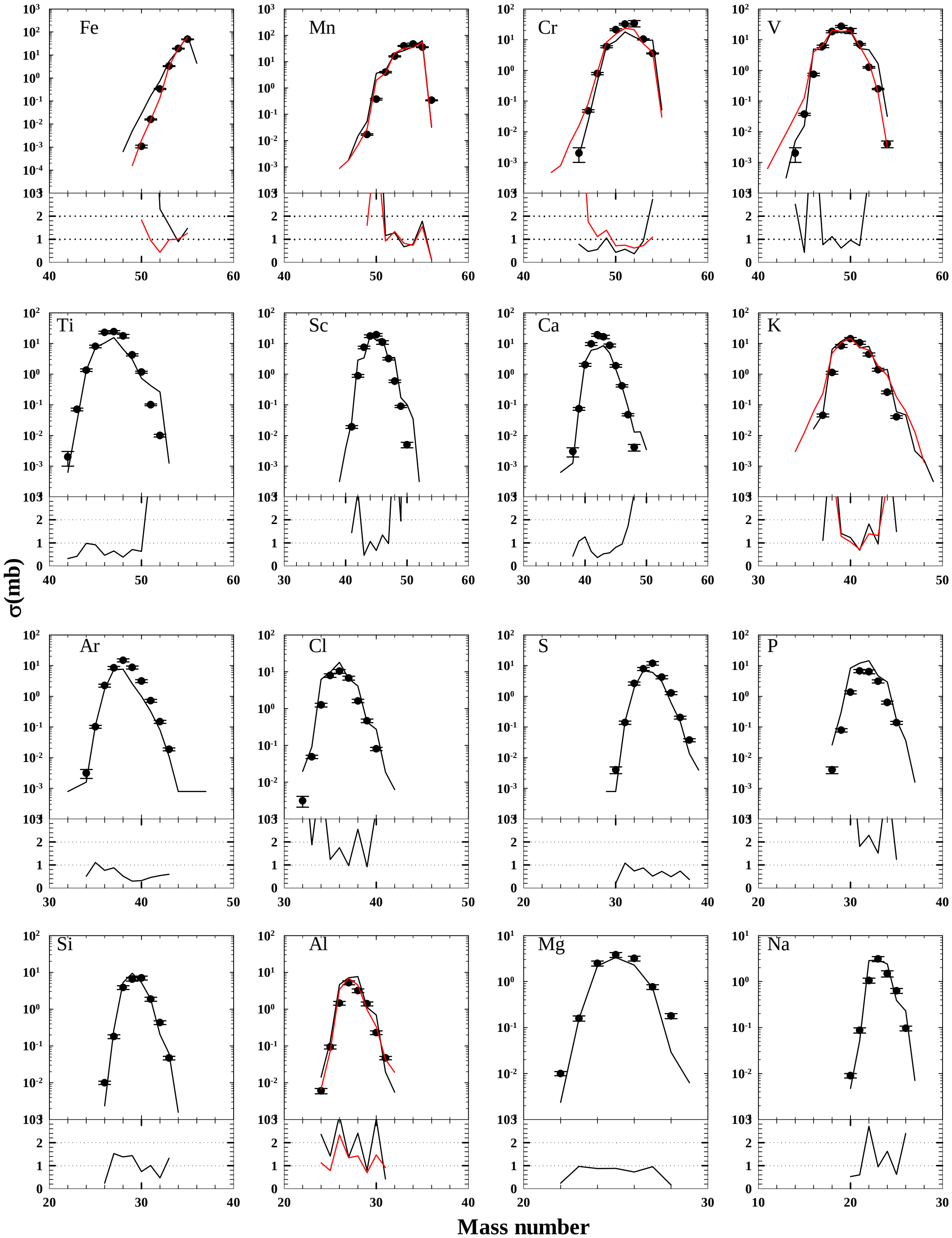
P_{pre}=1 or 0

Now

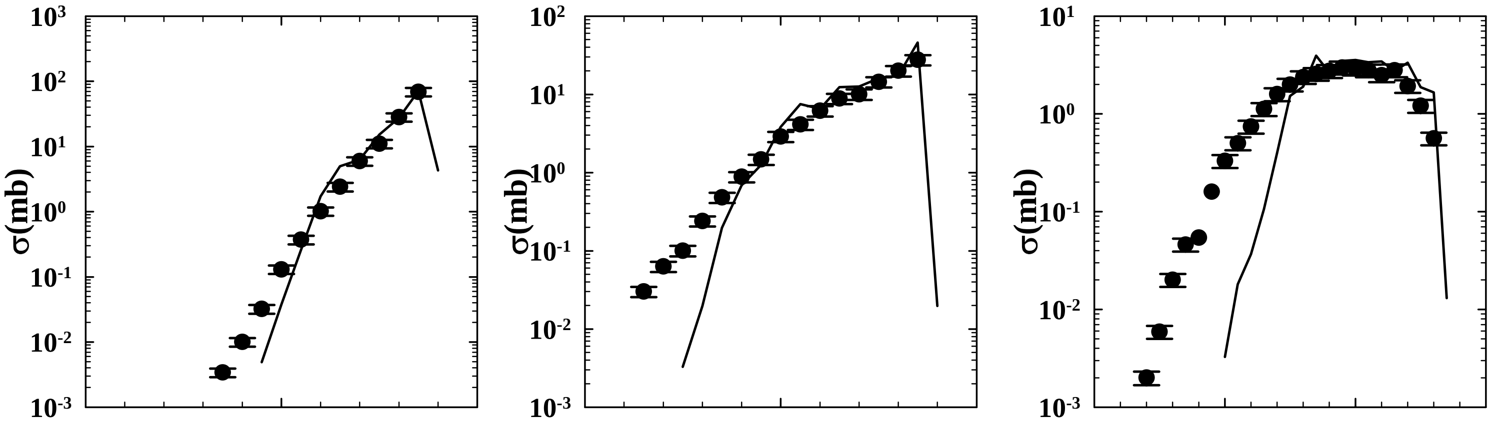
$$P_{pre}(n/n_{eq}) = 1 - \exp(-\frac{(n/n_{eq}-1)^2}{2\sigma_{pre}^2})$$

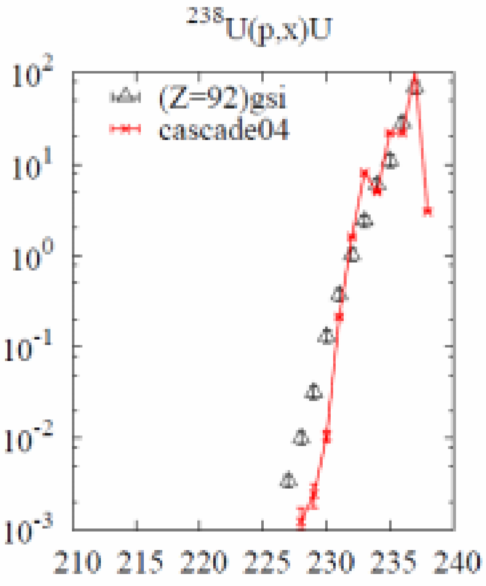
σ_{pre}=0.22 (0.4 in CEM)

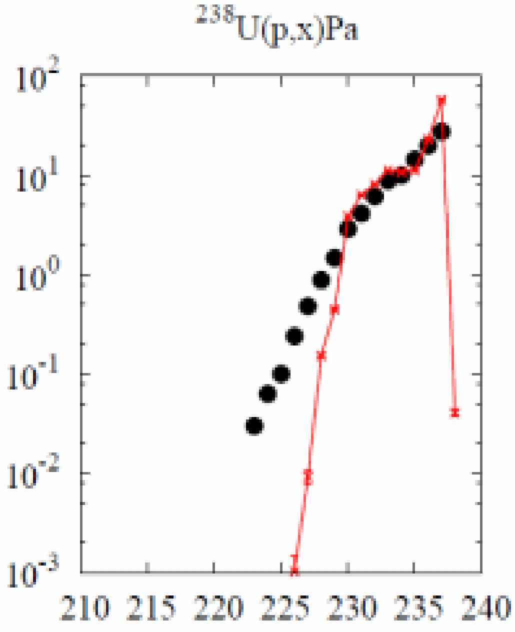
P (1GeV)+^{56}Fe

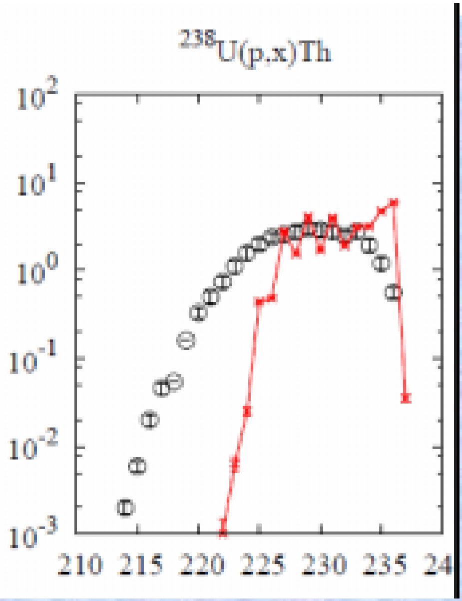


P (1GeV)+^{238}U

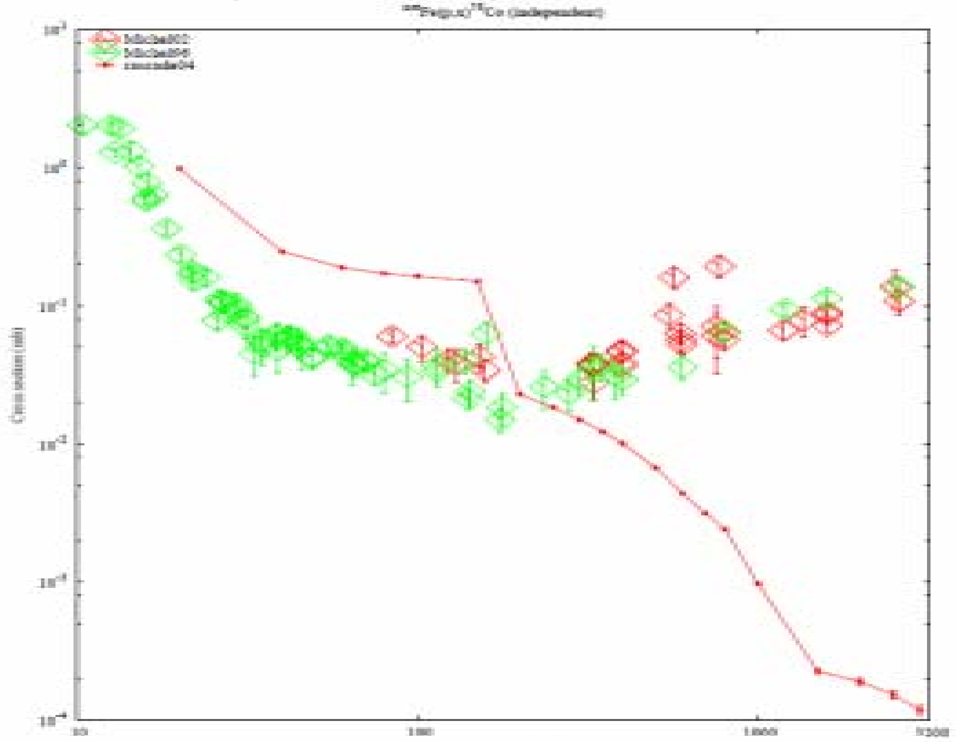

σ(mb)
10³
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
σ(mb)
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
σ(mb)
10¹
10⁰
10⁻¹
10⁻²
10⁻³

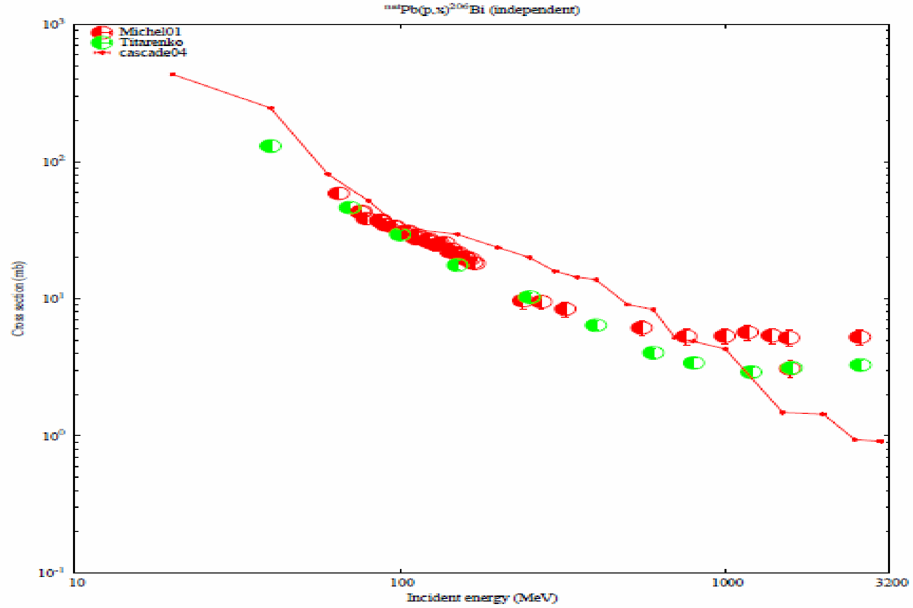

238U(p,x)U
(Z=92)gsi
cascade04
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
210 215 220 225 230 235 240


238U(p,x)Pa
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
210 215 220 225 230 235 240

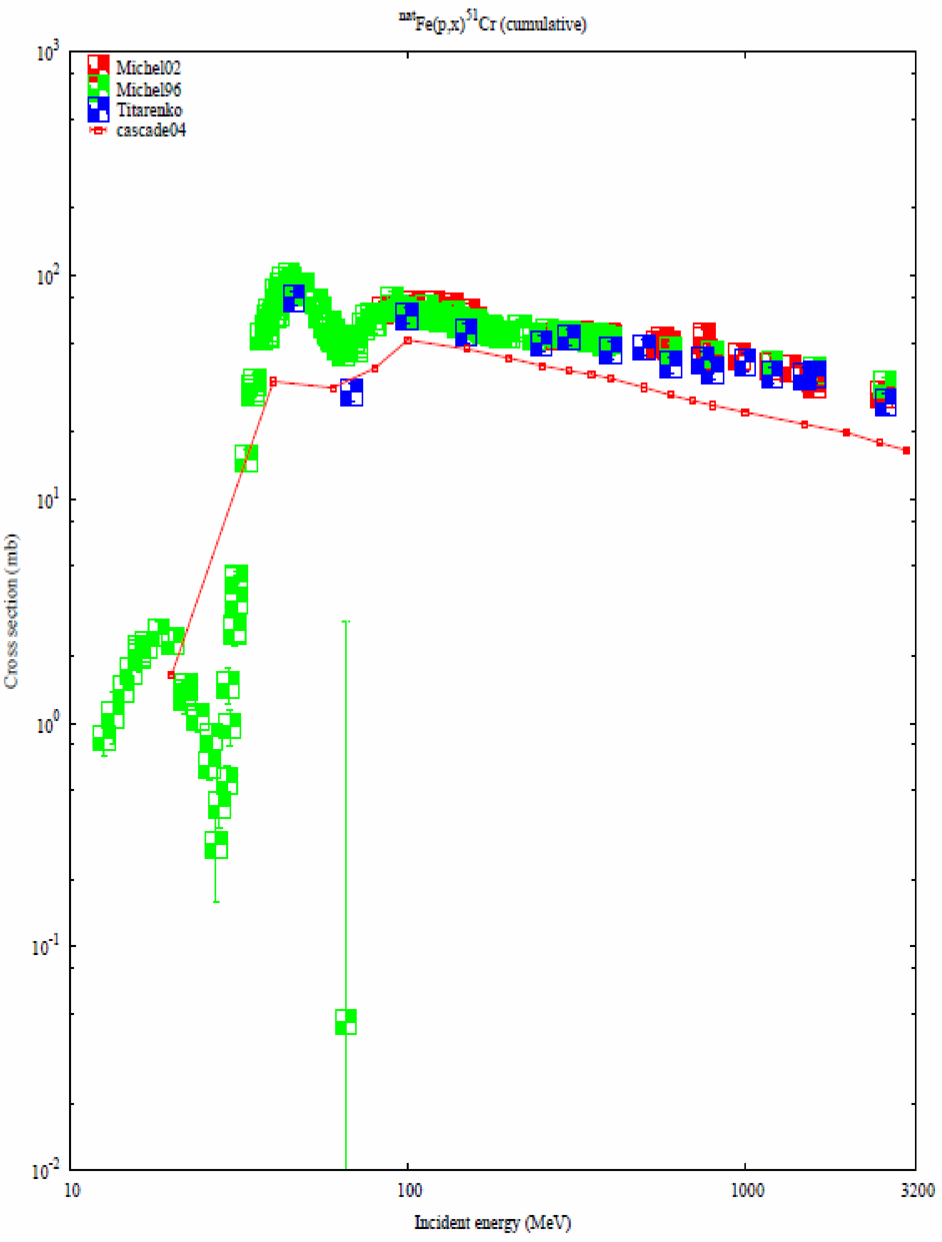

238U(p,x)Th
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
210 215 220 225 230 235 24

Charge exchange

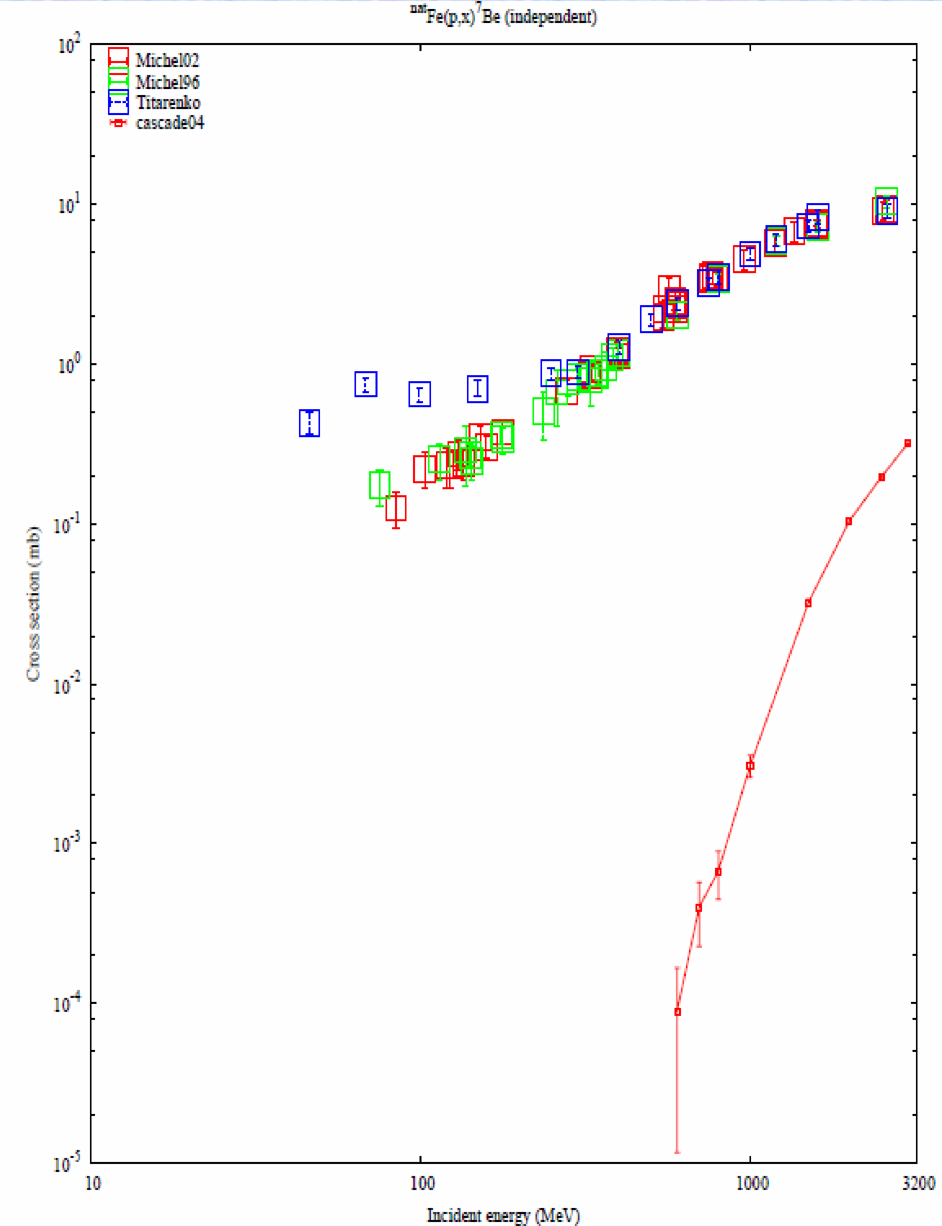




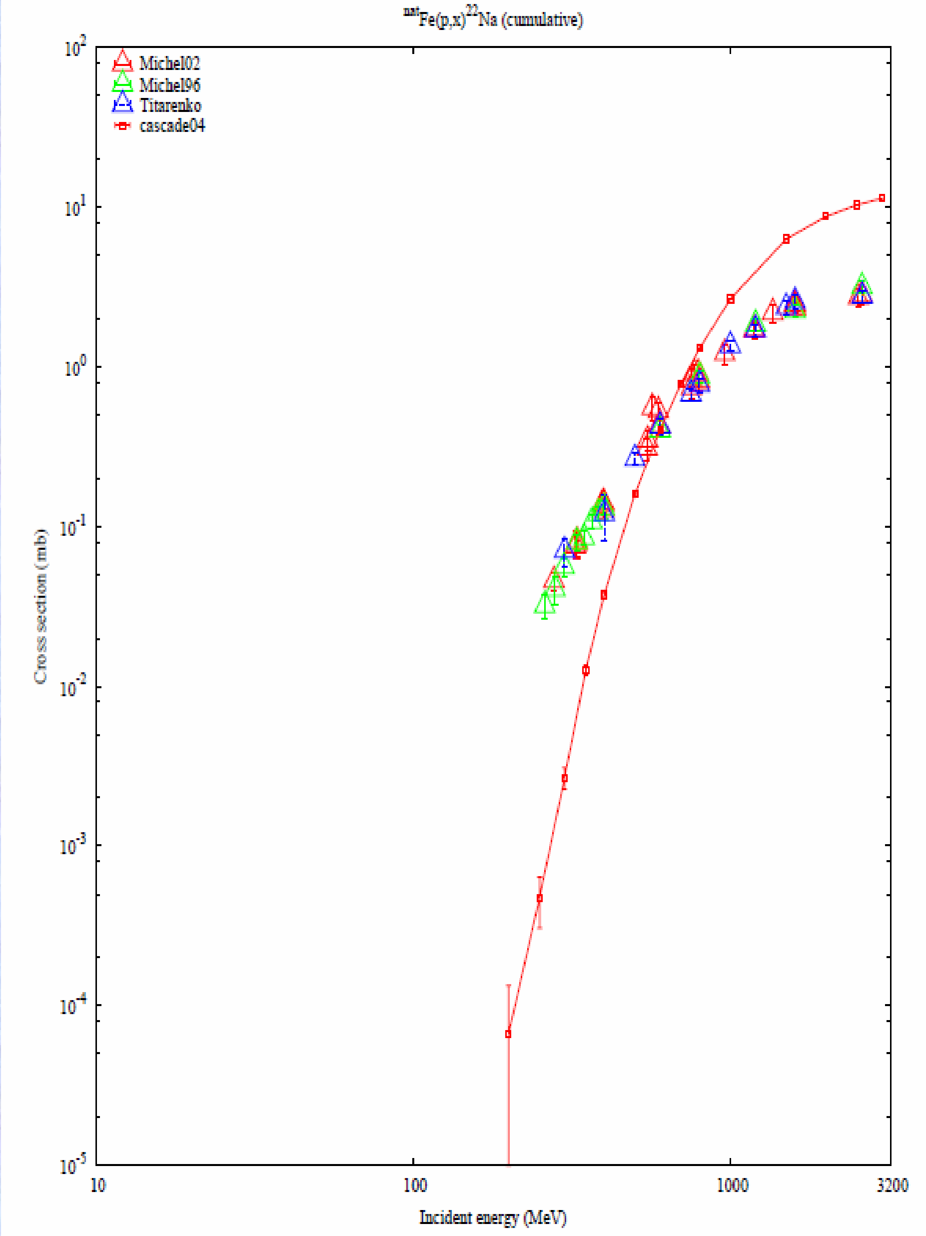
Close to target

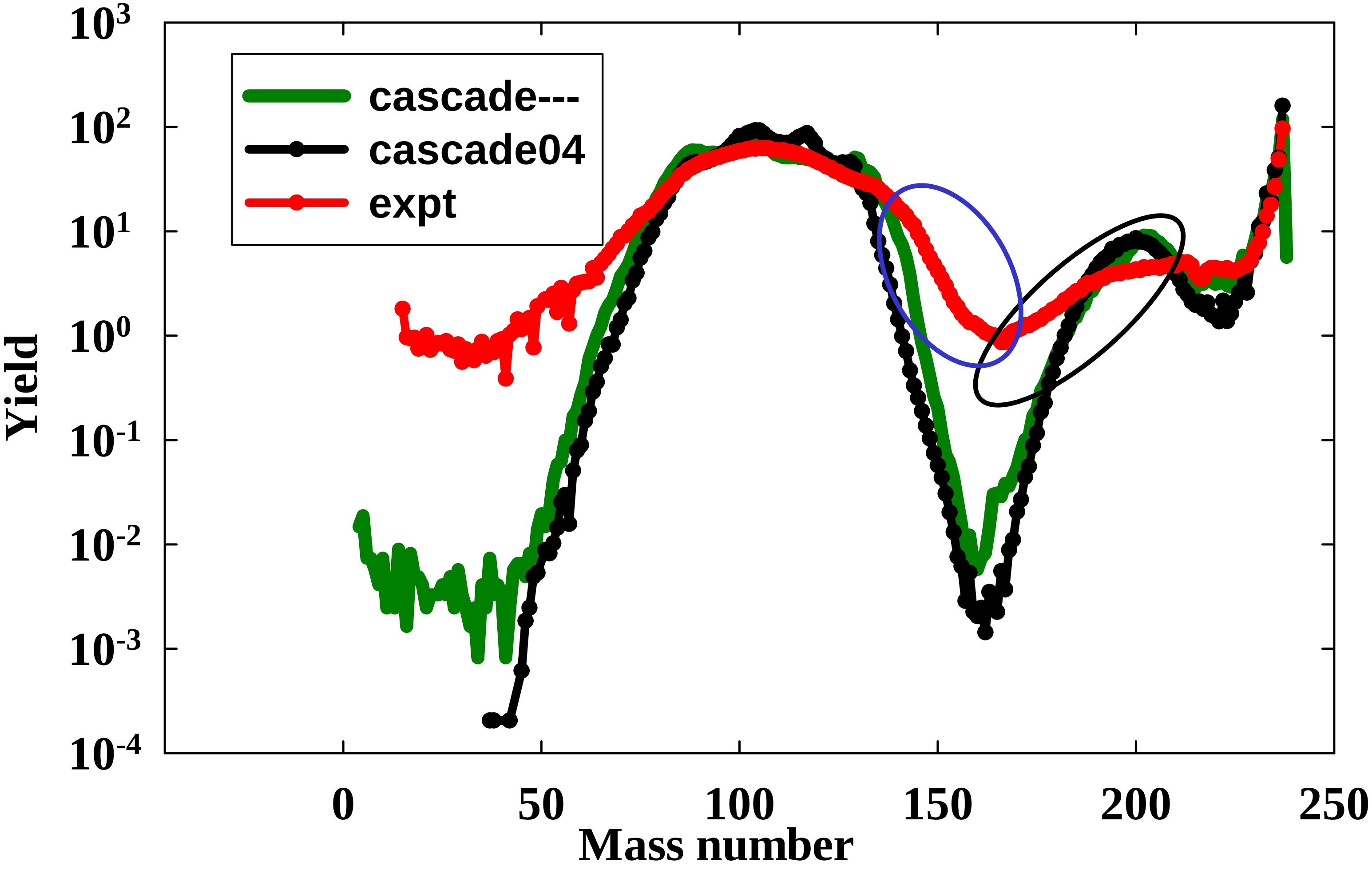


Light fragments



Far end of the target





10³
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
Yield
0
50
100
150
200
250
Mass number
cascade---
cascade04
expt

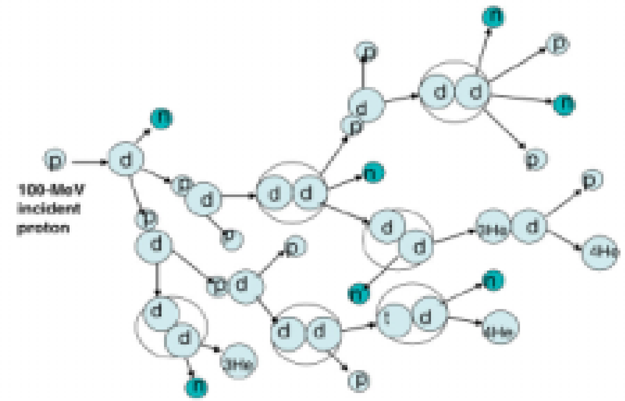


Fig. 1. A hypothetical cascade of breakup, fusion, and direct reactions initiated by a single proton. The cascade includes most of the reactions possible, although seldom will a proton induce the production of so many neutrons. To be competitive with spallation, on average two neutrons must be produced per 100-MeV proton. The reactions appear bunched together for illustrative purposes, but the proton momentum will carry most reaction particles strongly in the forward direction except for the energy-releasing fusion reactions.

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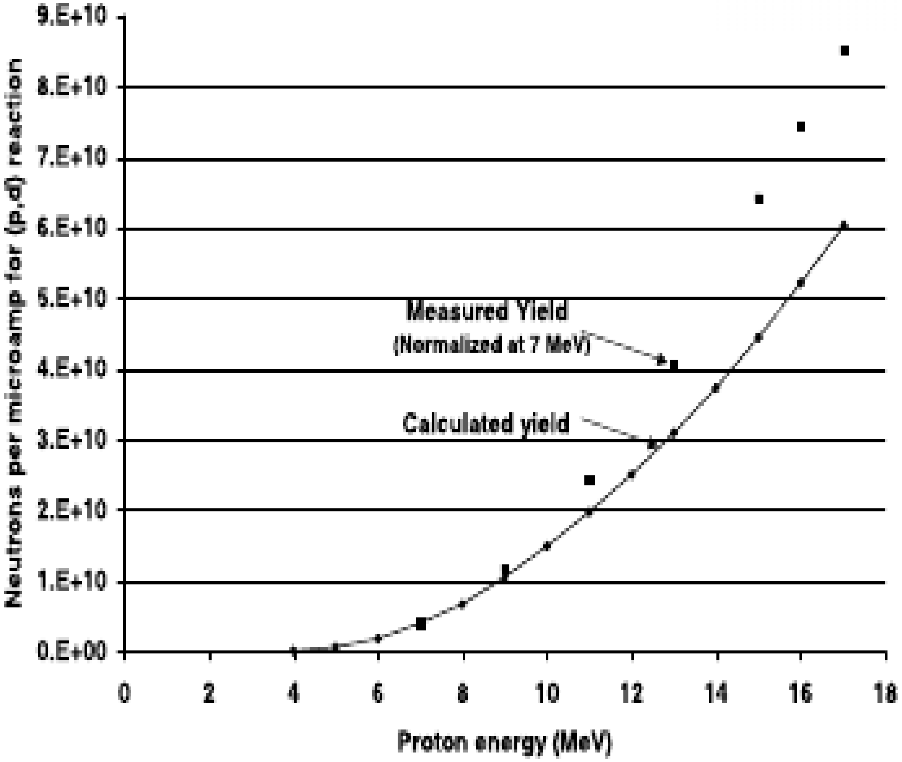

Fig. 3. Yield of neutrons from protons on a stopping-length deuterium target. The line through the lower set of points shows the expected yield based only on the $p + d \rightarrow n + 2p$ reaction. The upper points in the 7- to 17-MeV range are the measured points. The measured yield is therefore increasing faster than from the calculated reaction alone, indicating the substantial enhancement already present from fusion and other reactions in the energy range of this experiment.

धन्यवाद

Thank you for your attention