Zentrum für Strahlenschutz und Radioökologie
Leibniz Universität Hannover

# Benchmark of Spallation Models

# Results of a global analysis: Residues

Rolf Michel

Second Advanced Workshop on
Model Codes for Spallation Reactions,
Saclay, February 8 - 11, 2010

# Content

# Modell and Code Intercomparisons

**International Code Comparison for Intermediate Energy Nuclear Data,** M. Blann, H. Gruppelaar, P. Nagel, J. Rodens, NEA/OECD, NSC/DOC(94)-2, Paris 1993

**International Codes and Model Intercomparison for Intermediate Energy Activation Yields,** R. Michel, P. Nagel, NEA/OECD, NSC/DOC(97)-1, Paris 1997

# Participation in Intercomparison by Code Name and Physics Employed

**1993**

| Code Name | Physics |
|---|---|
| HECC/MECC7 + EVAP-F | INC+EVAP |
| GEANT | INC+EVAP |
| HERMES (HETC - KFA2) | INC+PE+EVAP |
| LAHET | INC+PE+EVAP |
| LAHET | INC+PE+EVAP |
| HETC-3 STEP | INC+PE+EVAP |
| CEM92M | INC+PE+EVAP |
| CEM92 | INC+PE+EVAP |
| NUCLEUS | INC+EVAP+FERMI STATISTICS |
| ALICE92 | PE+EVAP |
| ALICE87 MOD | PE+EVAP |
| ALICE F | PE+EVAP |
| PEQAQ2 | PE (EVAP VIA MASTER EQ) |
| GNASH | EXCITON+HAUSER-FESHBACH EVAP |
| FKK-GNASH | FKK+EXCITON +EVAP (H-F) |
| KAPSIES+GRAPE | FKK+EVAP |
| QMD | INC+2 Body Forces Between Collisions |
| SYSTEMATICS | SYSTEMATICS |

# Participation in Intercomparison by Code Name and Physics Employed

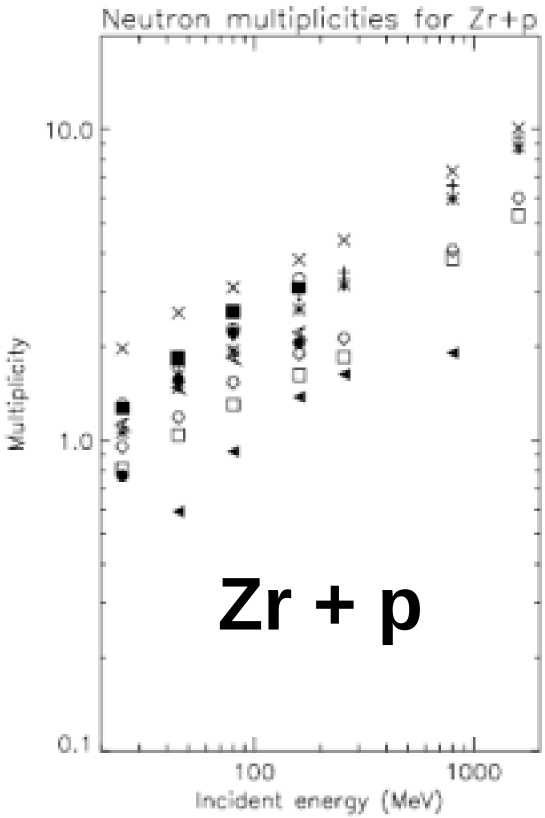
# 1997

| code used | physical model employed |
|---|---|
| PEQAG2 (extended) | PE + EVAP via MASTER EQ. |
| ALICE 92 | PE + EVAP |
| HMS-ALICE | HMS + EVAP |
| FKK-GNASH | FKK+EXCITON+HAUSER FESHBACH EVAP |
| QMDRELP+SDMRELP | QMD + SDM |
| HET/BRUYERE | INC + EVAP |
| PACE + MSM | INC + MSM |
| ISABEL-EVA | INC + EVAP |
| AREL | PE + EVAP (GDH) |
| HETC-FRG | INC + PE + EVAP + FRAGMENTATION |
| INUCL | INC + EVAP |
| MINGUS | FKK + EVAP |
| ISABEL/SMM | INC + SMM + EVAP |
| CEM 95 | INC + PE + EVAP |
| HET-KFA2 | INC + EVAP |
| SPALL (modified)/YIELD | TSAO & SILBERBERG SYSTEMATICS |
| ALICE -IPPE | PE + EVAP |
| CASCADE | INC + EVAP |
| DISCA | INC + EVAP |
| MSDM | INC + PE + SMM + EVAP + FERMI BREAKUP |
| HETC-3STEP | INC + PE + EVAP |
| MECC7 + EVAP_F | INC + EVAP |

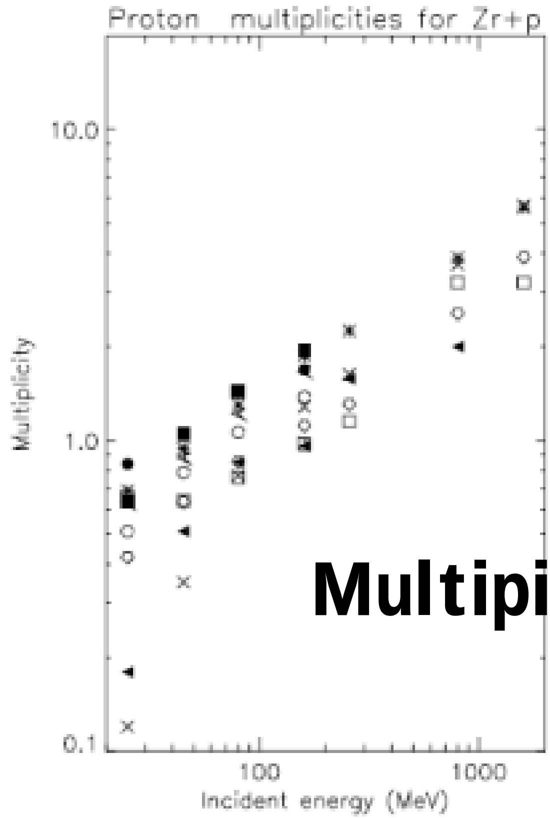
**International Code Comparison**
**for Intermediate Energy Nuclear Data**
**M. Blann et al. ( 1993)**

# Double differential cross sections, (p,*x*n) and (p,*x*p) 20 MeV – 1600 MeV, Zr-90 and Pb-208

| E incident | Angles |
|---|---|
| 25 MeV | 0° to 180° in 20° increments |
| 45 MeV | 0° to 180° in 20° increments |
| 80 MeV | 0°, 11°, 25°, 45°, 69°, 95°, 120°, 145°, 180° |
| 160 MeV | 0°, 11°, 25°, 45°, 69°, 95°, 120°, 145°, 180° |
| 256 MeV | 0°, 7.5°, 30°, 60°, 120°, 150°, 180° |
| 800 MeV | 0°, 7.5°, 30°, 60°, 120°, 150°, 180° |
| 1600 MeV | 0°, 7.5°, 30°, 60°, 120°, 150°, 180° |

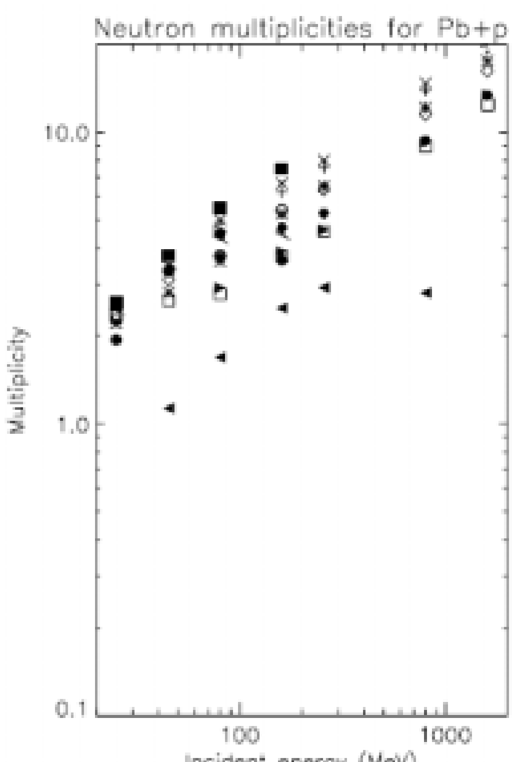


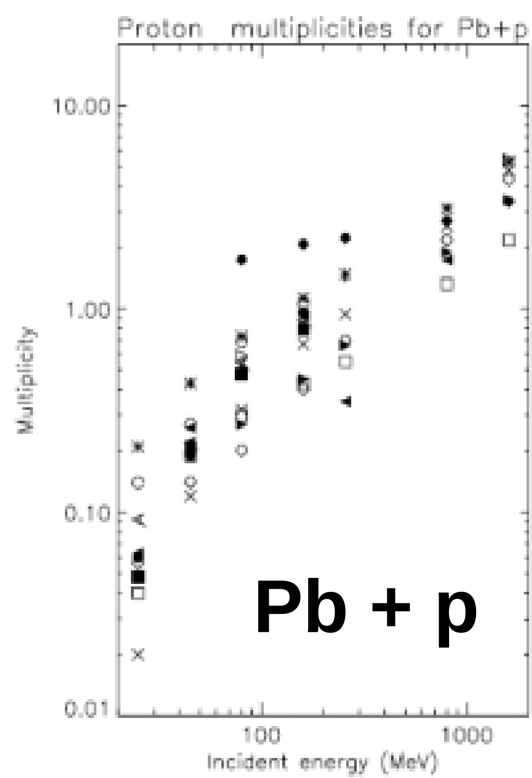

Zr + p




**Multipicities**

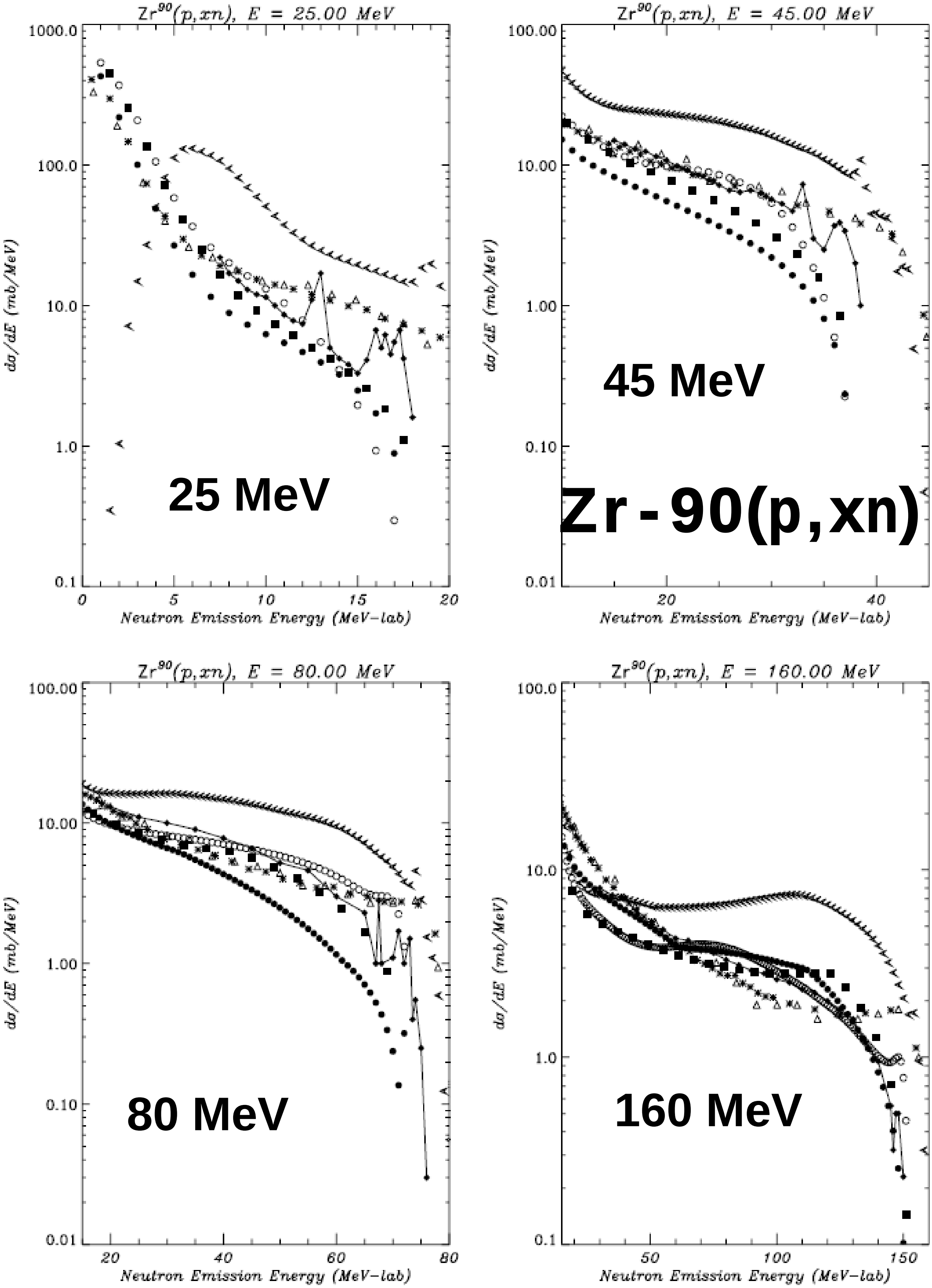
**What is a factor of two among friends?**



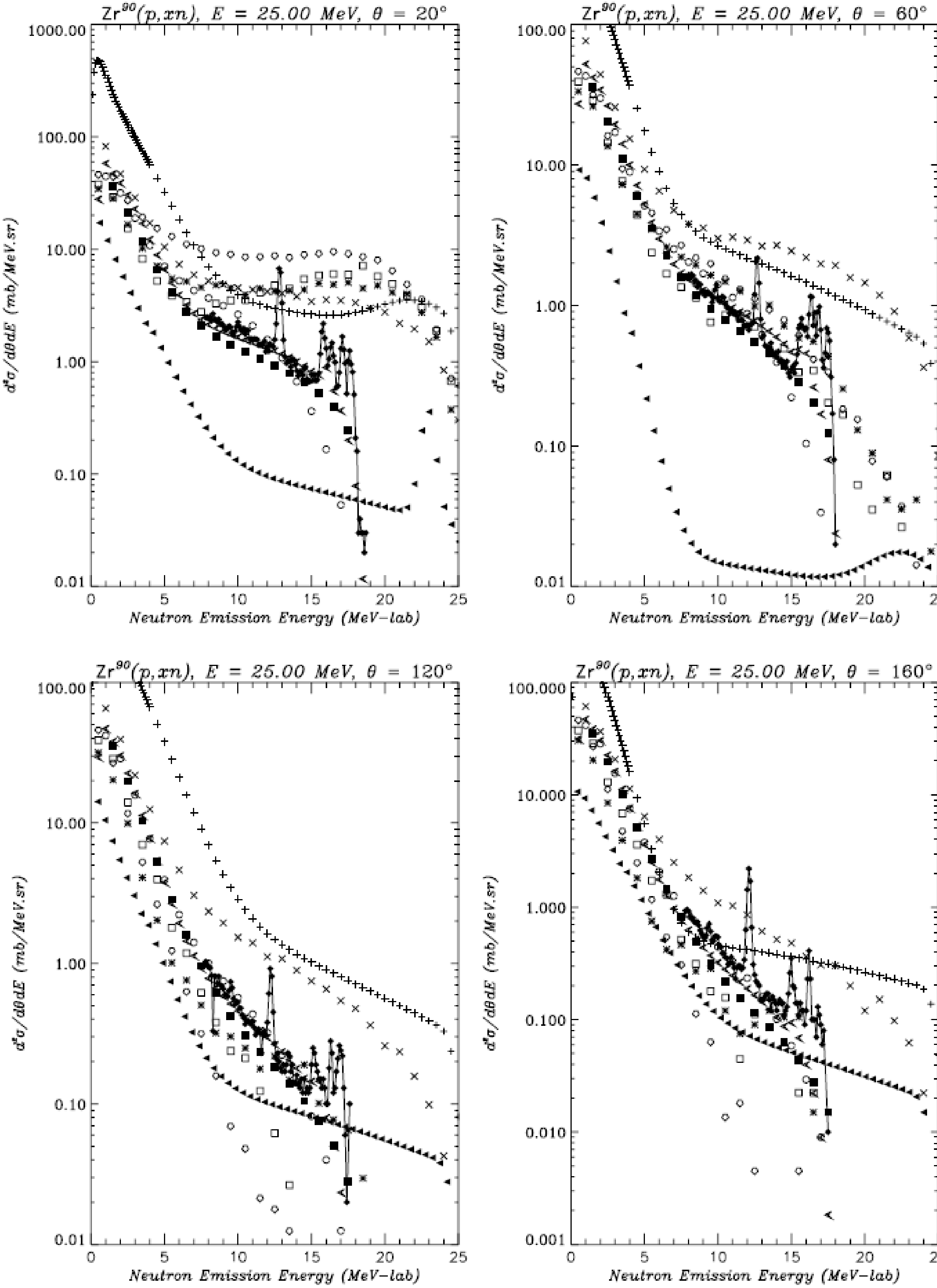

Pb + p

**NSC/DOC(94)-2**

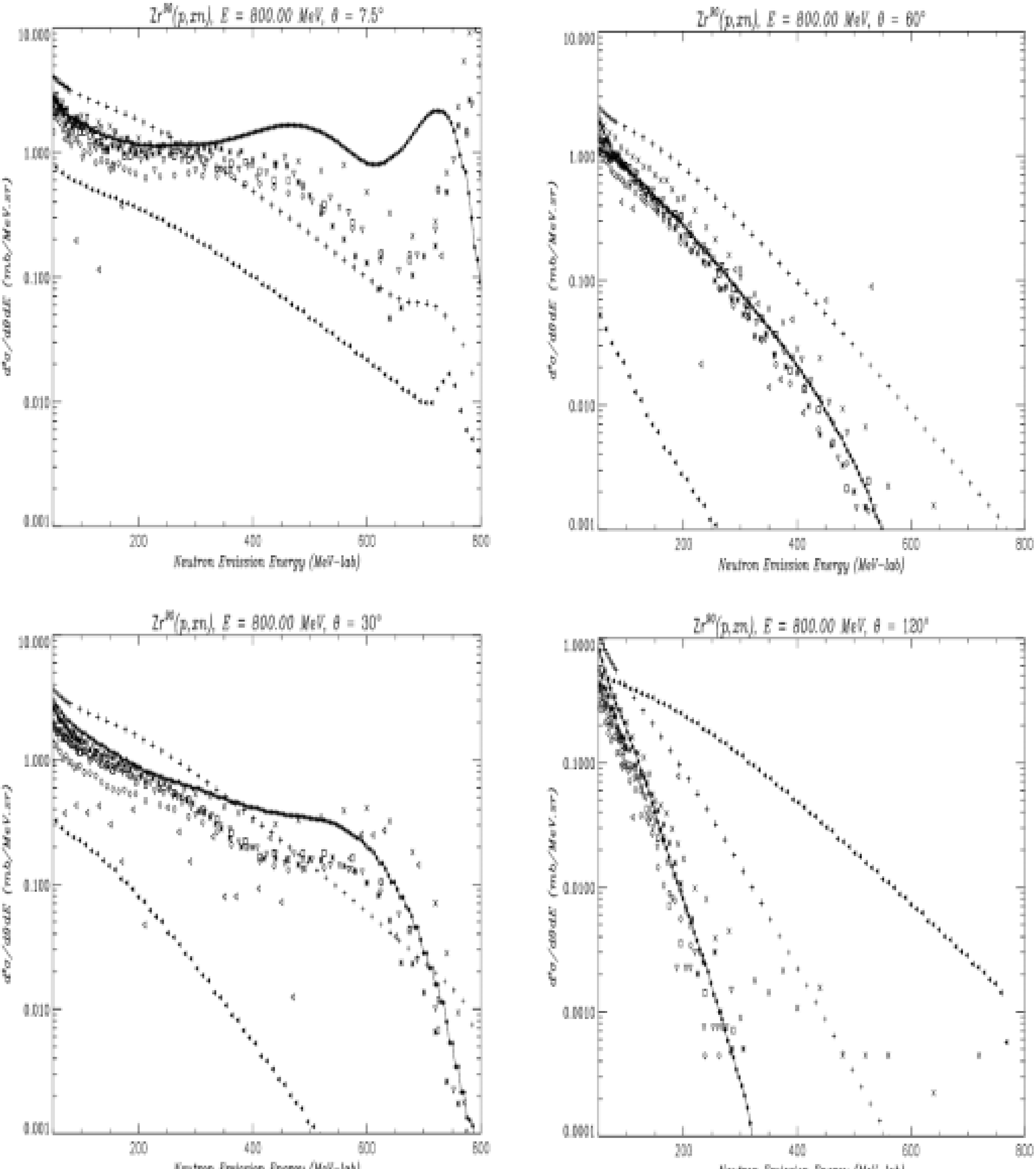

Zr$^{90}$(p,xn), E = 25.00 MeV
25 MeV
Zr$^{90}$(p,xn), E = 45.00 MeV
45 MeV
Zr-90(p,xn)
Zr$^{90}$(p,xn), E = 80.00 MeV
80 MeV
Zr$^{90}$(p,xn), E = 160.00 MeV
160 MeV
dσ/dE (mb/MeV)
Neutron Emission Energy (MeV-lab)

NSC/DOC(94)-2

# Zr-90(p,xn) @ 25 MeV

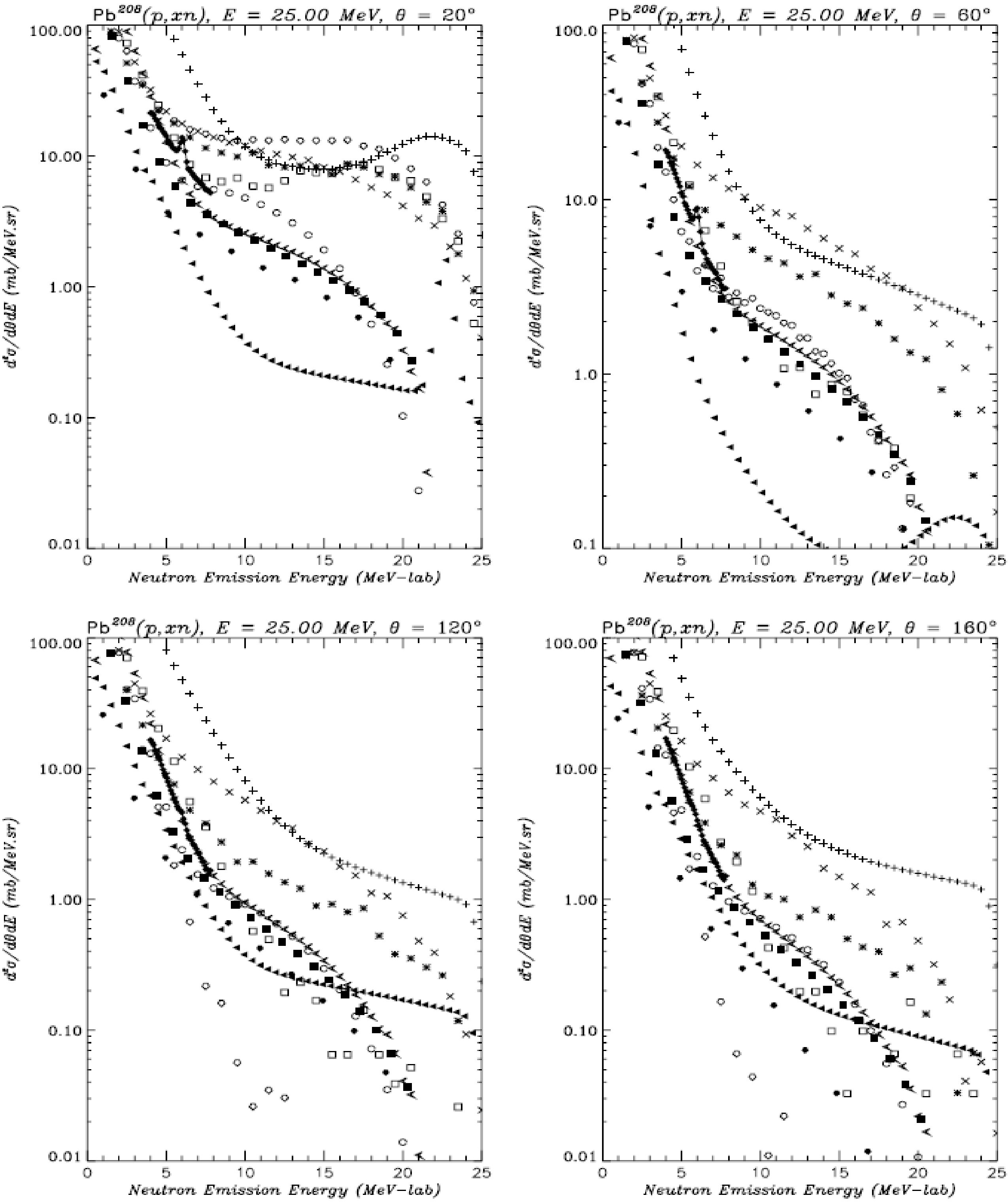


# Zr-90(p,xn) @ 800 MeV



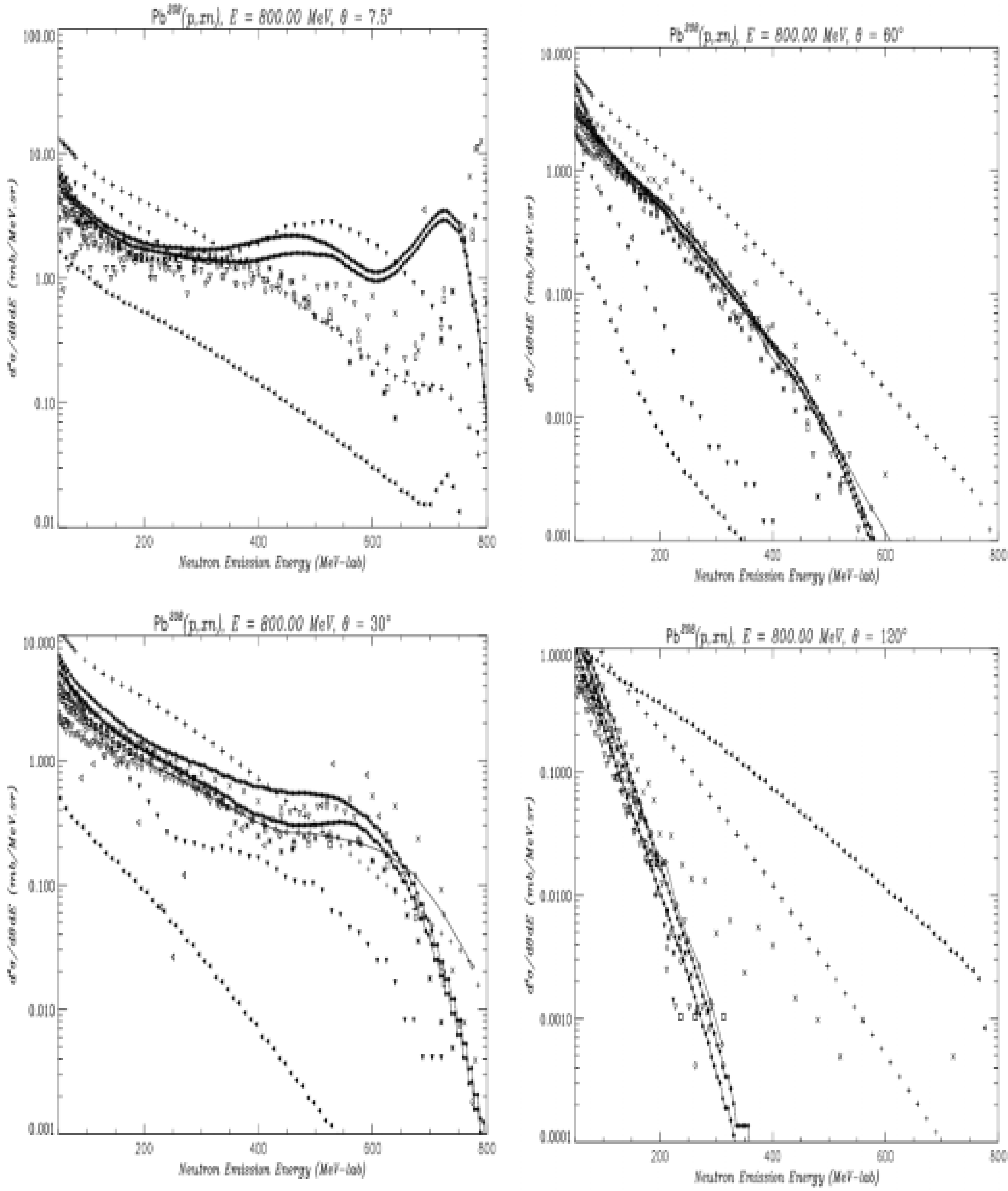

NSC/DOC(94)-2

# Pb-208(p,xn) @ 25 MeV




NSC/DOC(94)-2

# Pb-208(p,xn) @ 800 MeV




NSC/DOC(94)-2

# Conclusion

**This exercise has, as a main goal, the display of results of model calculations versus high quality experimental data. Conclusions of such comparisons are subjective in nature. We have tried to attach a quite crude figure of merit for each entry at each energy and angle ....**

**From this exercise we may conclude that there is room for improvement in all codes, and that modelling calculations on a predictive basis may have uncertainties of the order of +50%.**

**…**

**The codes tested herein do well in reproducing many aspects of the microscopic nuclear physics, i.e., the DDCS. There is room for improvement and such efforts would be well spent given the importance of IEND for future technological development.**

**NSC/DOC(94)-2**

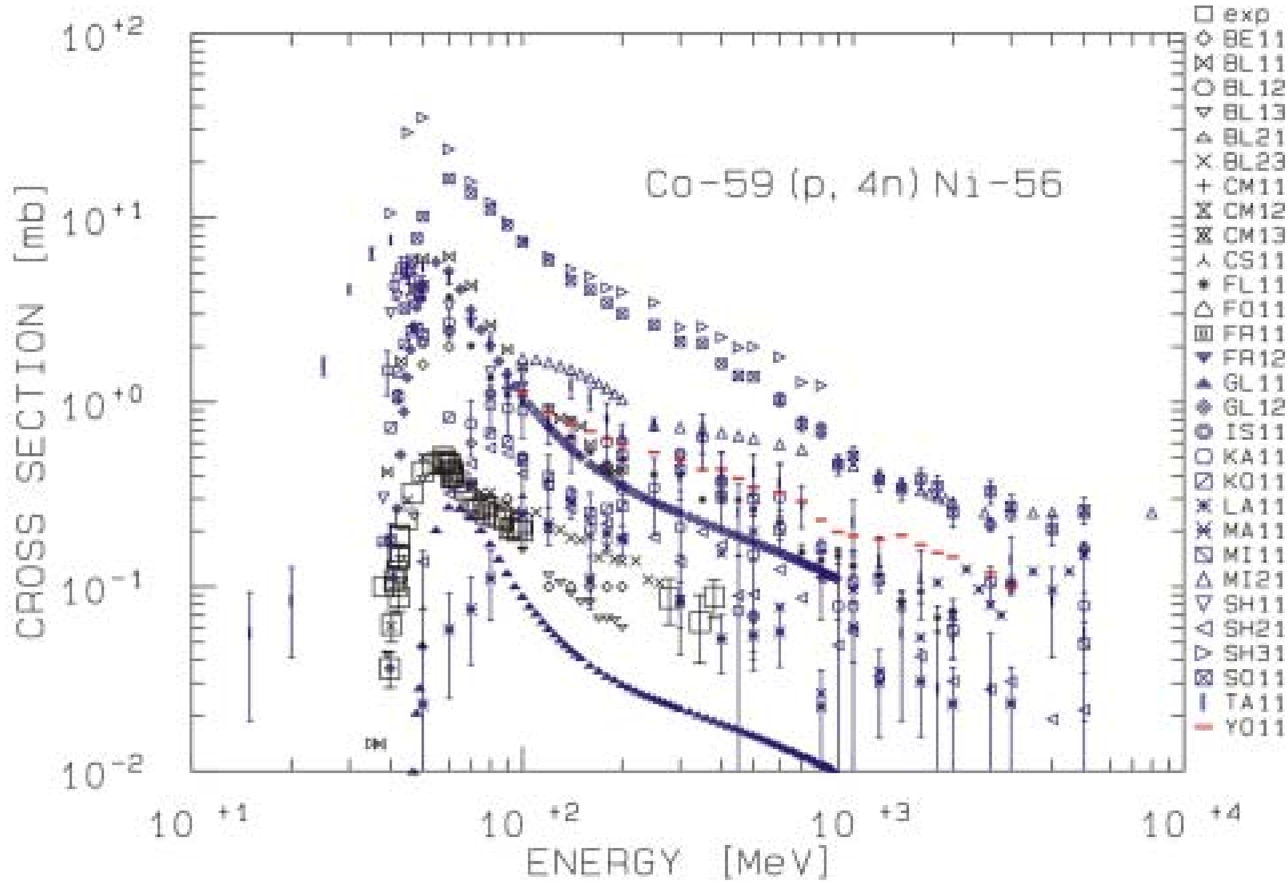
# International Codes and Model Intercomparison for Intermediate Energy Activation Yields

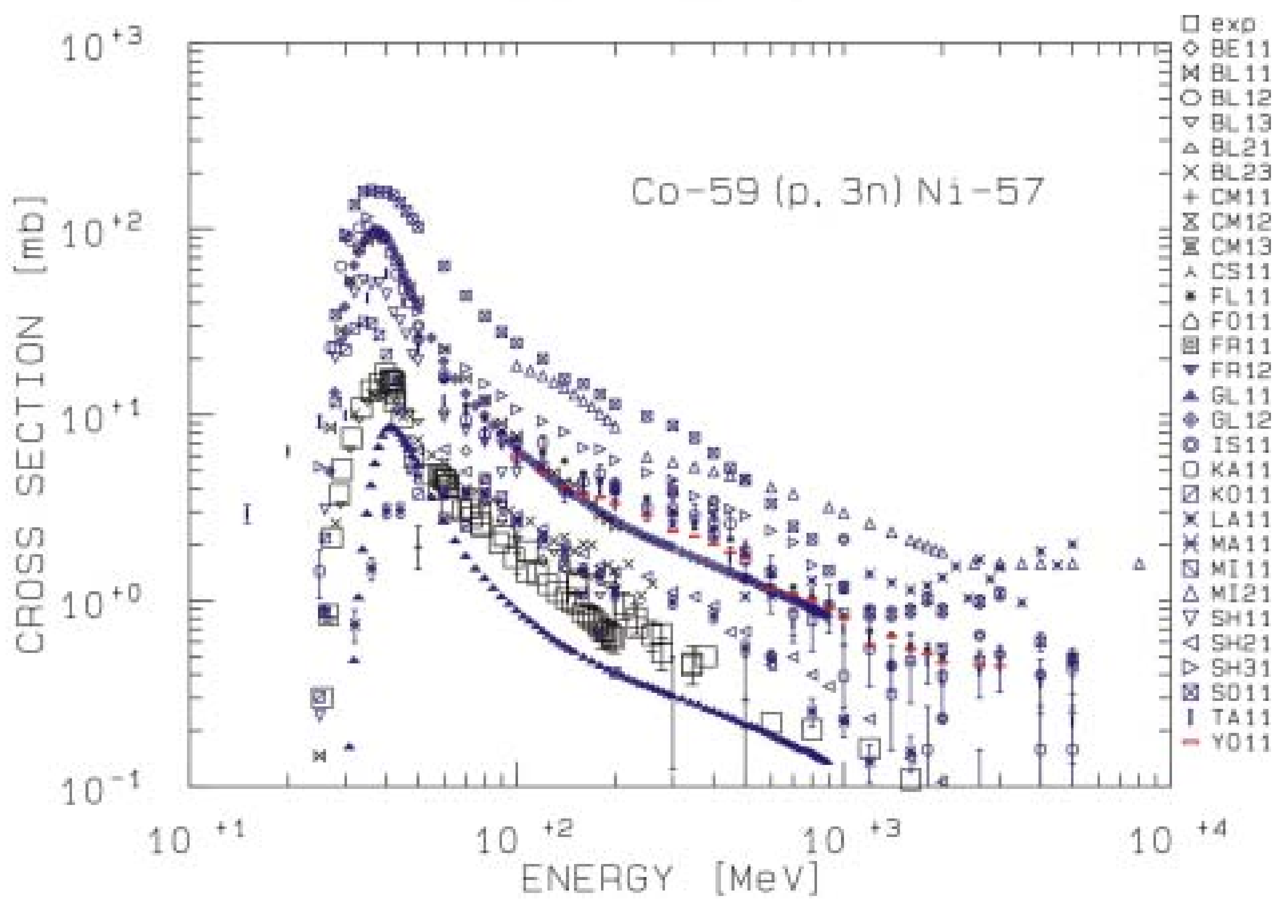
**1997**

**Blind intercomparison: E < 5 GeV**
**Target elements: O, Al, Fe, Co, Zr, Au**

| code used | physical model employed |
|---|---|
| PEQAG2 (extended) | PE + EVAP via MASTER EQ. |
| ALICE 92 | PE + EVAP |
| HMS-ALICE | HMS + EVAP |
| FKK-GNASH | FKK+EXCITON+HAUSER FESHBACH EVAP |
| QMDRELP+SDMRELP | QMD + SDM |
| HET/BRUYERE | INC + EVAP |
| PACE + MSM | INC + MSM |
| ISABEL-EVA | INC + EVAP |
| AREL | PE + EVAP (GDH) |
| HETC-FRG | INC + PE + EVAP + FRAGMENTATION |
| INUCL | INC + EVAP |
| MINGUS | FKK + EVAP |
| ISABEL/SMM | INC + SMM + EVAP |
| CEM 95 | INC + PE + EVAP |
| HET-KFA2 | INC + EVAP |
| SPALL (modified)/YIELD | TSAO & SILBERBERG SYSTEMATICS |
| ALICE -IPPE | PE + EVAP |
| CASCADE | INC + EVAP |
| DISCA | INC + EVAP |
| MSDM | INC + PE + SMM + EVAP + FERMI BREAKUP |
| HETC-3STEP | INC + PE + EVAP |
| MECC7 + EVAP_F | INC + EVAP |

R. Michel, P. Nagel, NSC/DOC(97)-1, NEA/OECD, Paris, 1997




**R. Michel, P. Nagel, NSC/DOC(97)-1, NEA/OECD, Paris, 1997**

# Deviation Factors
## as means of descriptive statistics

**Define a mean square logarithmic deviation:**

$$< (\log(\sigma_{\exp}) - \log(\sigma_{\text{theo}}))^2 >$$
$$= \sum_i (\log(\sigma_{\exp,i}) - \log(\sigma_{\text{theo},i}))^2 / NS$$

**Average deviation factor for each reaction:**

$$< F >$$
$$= 10^{\sqrt{<(\log(\sigma_{\exp})-\log(\sigma_{\text{theo}}))^2>}}$$

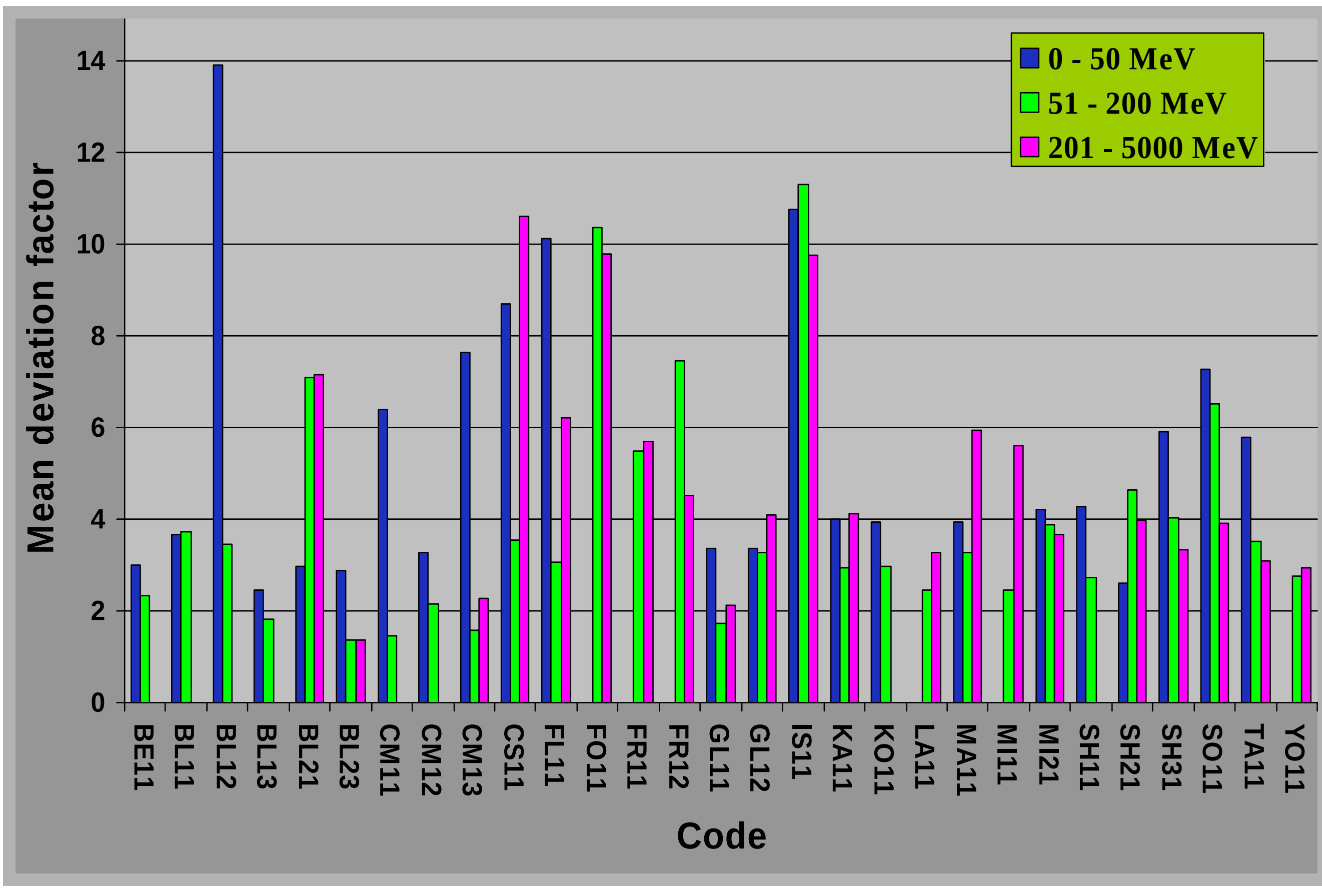
**Global mean deviation factor :**

$$<< F >>$$
$$= 10^{\sqrt{<<(\log(\sigma_{\exp})-\log(\sigma_{\text{theo}}))^2>>}}$$

R. Michel, P. Nagel, NSC/DOC(97)-1, NEA/OECD, Paris, 1997

**R. Michel, P. Nagel, International Codes and Model Intercomparison for Intermediate Energy Activation Yields, NSC/DOC(97)-1, NEA/OECD, Paris, 1997**

# Conclusion




**On the average,**

- **predictions are at best within a factor of two;**
- **discrepancies reach more than a factor of ten.**

**Different predictions for individual reactions easily cover two orders of magnitude.**

# Goals of the present intercomparison

- to assess the prediction capabilities of the spallation models used or that could be used in the future in high-energy transport codes,
- to provide direct visual comparisons between data and calculation,
- to use quantitative measures such as Figures-of-Merit and deviation factors for the agreement between calculations and experimental data,
- to understand the reason for the success or deficiency of the models in the different mass and energy regions or for the different exit channels,
- to reach a consensus, if possible, on some of the physics ingredients that should be used in the models.

- It is not the goal of the intercomparison to define “the best” model but for each observable, for different energy or mass range and to give recommendations to use one model rather than another one

# Residual-nuclide-distribution cross sections measured in inverse kinematics by bombardment of hydrogen with heavy ions.

| beam | energy in A MeV | reference |
|---|---|---|
| Fe | 300 | C. Villagrasa-Canton et al., Phys. Rev. C 75 (2007) 044603 |
| Fe | 1000 | C. Villagrasa-Canton et al., Phys. Rev. C 75 (2007) 044603<br>P. Napolitani et al., Phys. Rev. C 70 (2004) 054607 |
| Pb | 500 | L. Audouin et al., Nucl. Phys. A768 (2006) 1 |
| Pb | 1000 | T. Enqvist et al., Nucl. Phys. A686 (2001) 481 |
| U | 1000 | J. Taieb et al., Nucl. Phys. A 724 (2003) 413<br>M. Bernas et al., Nucl. Phys. A 725 (2003) 213<br>M. Bernas et al., Nucl. Phys. A765 (2006) 197<br>M. V. Ricciardi et al., Phys. Rev. C 73 (2006) 014607 |

# Excitation functions of residual-nuclide-production measured in classical kinematics

| Tar-get | Energies in MeV | reference |
|---|---|---|
| Fe | 20 – 2600 | R. Michel et al., Nucl. Instr. and Meth. B 103 (1995) 183<br>Th. Schiekel, R. Michel et al., Nucl. Instr. and Meth. B 114 (1996) 91<br>R. Michel et al., Nucl. Instr. and Meth. B 129 (1997) 153<br>R. Michel et al., Nucl. Sci. Tech., Supplement 2 (2002) 242<br>K. Ammon, I. Leya et al., Nucl. Instr. and Meth. B 266 (2008) 2<br>Yu.E. Titarenko et al., PRC 78 (2008) 034615 |
| Pb | 20 – 2600 | M. Gloris et al., Nucl. Instr. and Meth. A463 (2001) 593<br>I. Leya et al., Nucl. Instr. and Meth. B229 (2005) 1<br>Y. E. Titarenko et al., Nucl. Instr. and Meth. A562 (2006) 801 |

# Some heretical considerations:

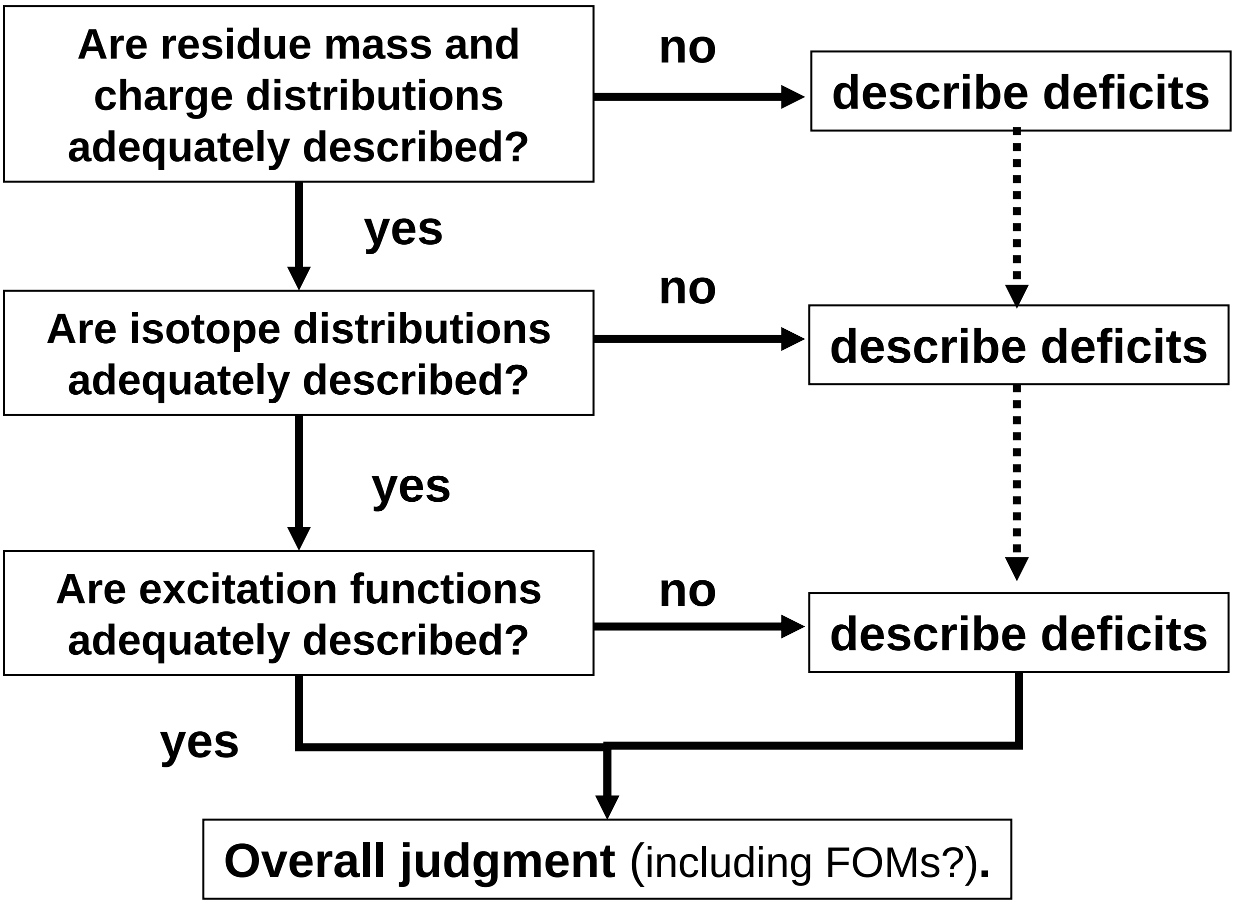
**Given the wealth of experimental information on residual nuclide production, we should be able to judge whether a model or code exhibits fundamental deficits in describing residual nuclide production.**

## Are such fundamental deficits potentially knock-out criteria for models and codes?

**Such criteria could be the complete failure to describe**

- **Mass distributions,**
- **Charge distributions,**
- **Isotopic distributions,**
- **Energy dependencies.**

# Method of a Systematic evaluation




## Categorization:

| low energies | | high energies | | |
|---|---|---|---|---|
| Fe 300 MeV | Pb 500 MeV | Fe 1 GeV | Pb 1 GeV | U 1 GeV |
| target near products | target near products | target near products | target near products | target near products |
| spallation products | spallation products | spallation products | spallation products | spallation products |
| light products | light products | light products | light products | light products |
| - | - | - | fission products | fission products |

# Rating System for Mass, Charge and Isotopic Distributions

**Categorization:**

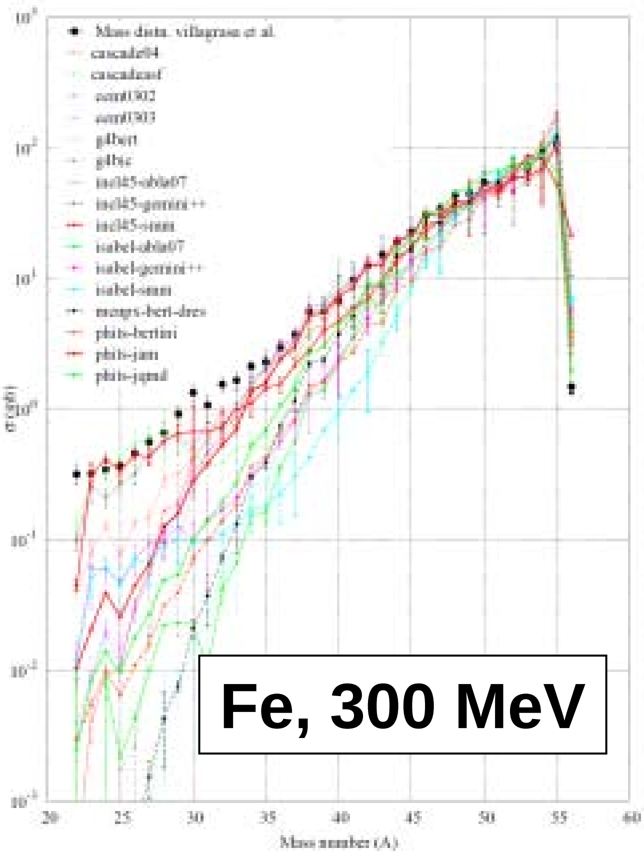
| low energies | | high energies | | |
|---|---|---|---|---|
| Fe 300 MeV | Pb 500 MeV | Fe 1 GeV | Pb 1 GeV | U 1 GeV |
| target near products | target near products | target near products | target near products | target near products |
| spallation products | spallation products | spallation products | spallation products | spallation products |
| light products | light products | light products | light products | light products |
| - | - | - | fission products | fission products |

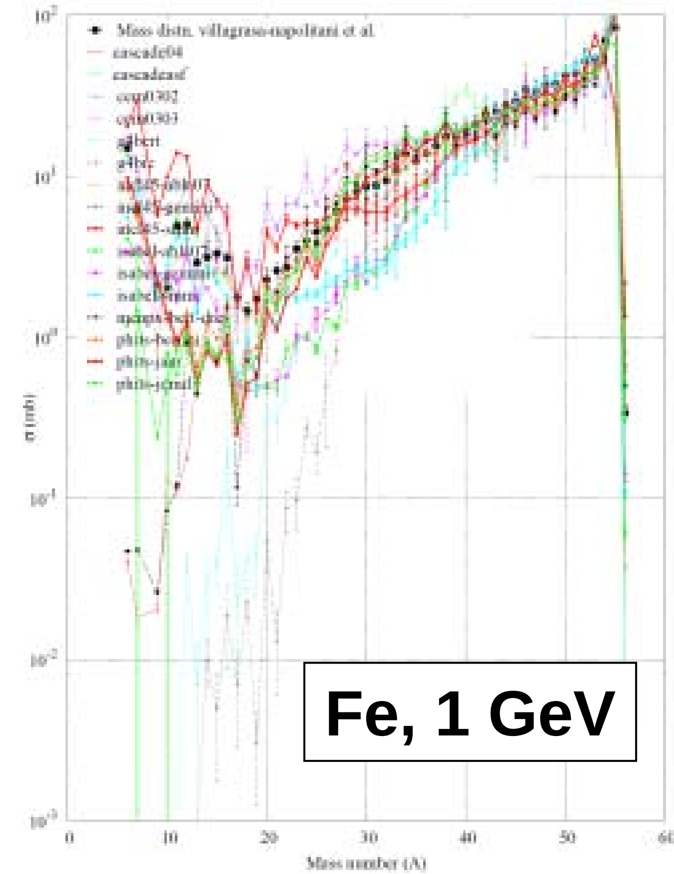
There are no data for light complex nuclei. Light complex particles can only be taken into account by looking for the excitation functions.

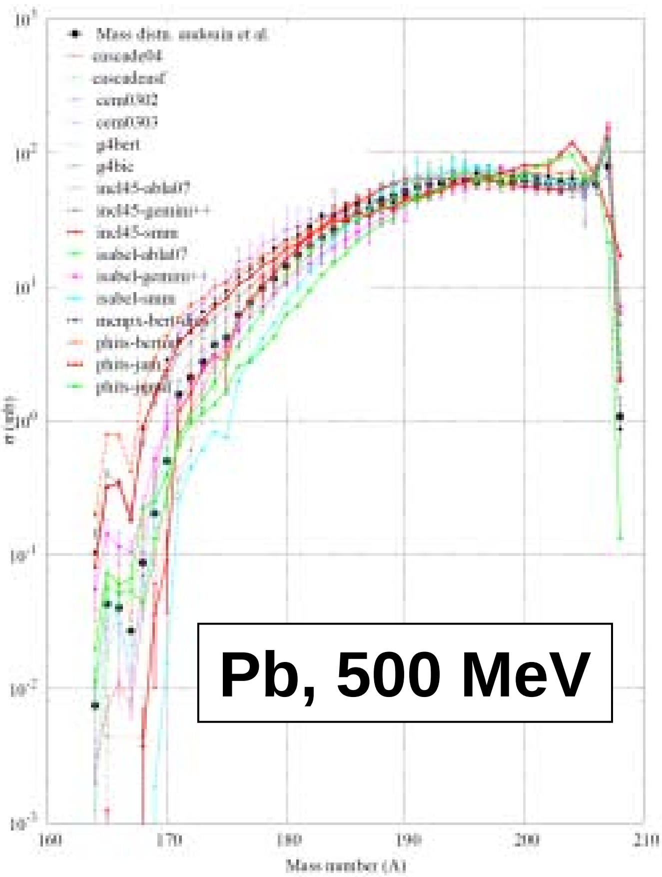
## Ratings:

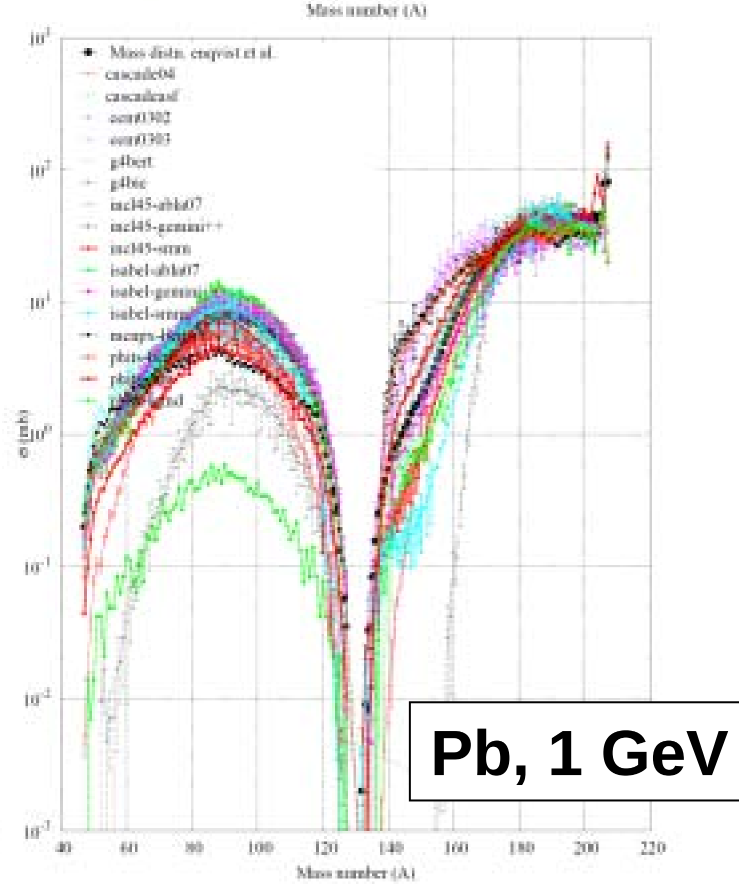
| | |
|---|---|
| 2 | good |
| 1 | moderately good , minor problems |
| -1 | moderately bad, particular problems |
| -2 | unacceptably bad, systematically wrong |

# Mass Distribution of Residues

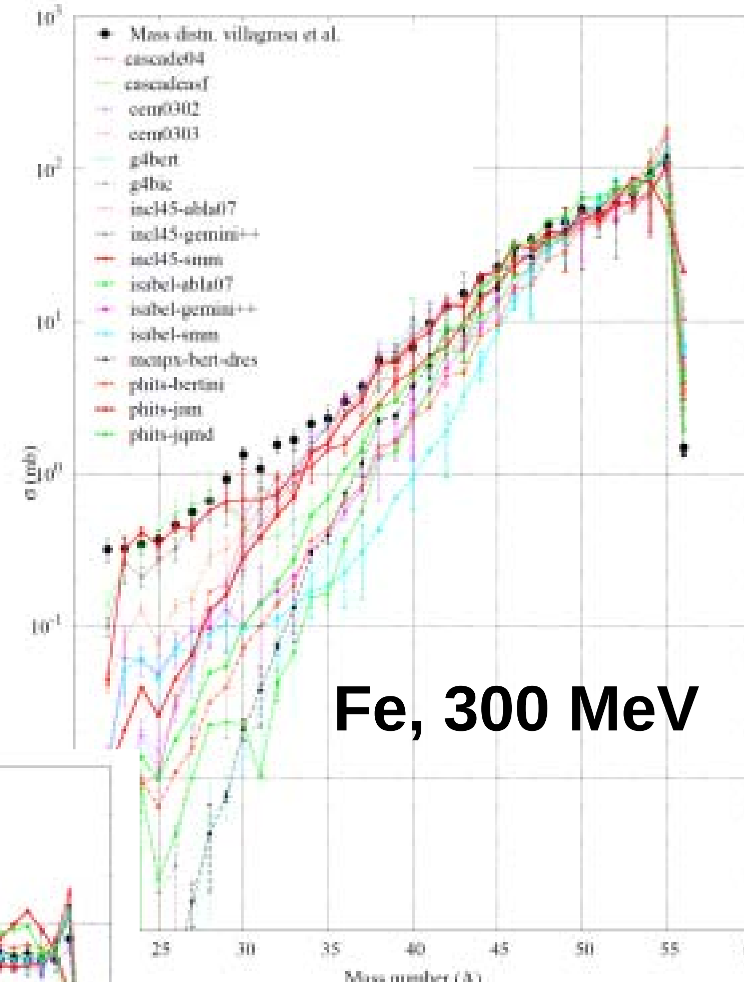


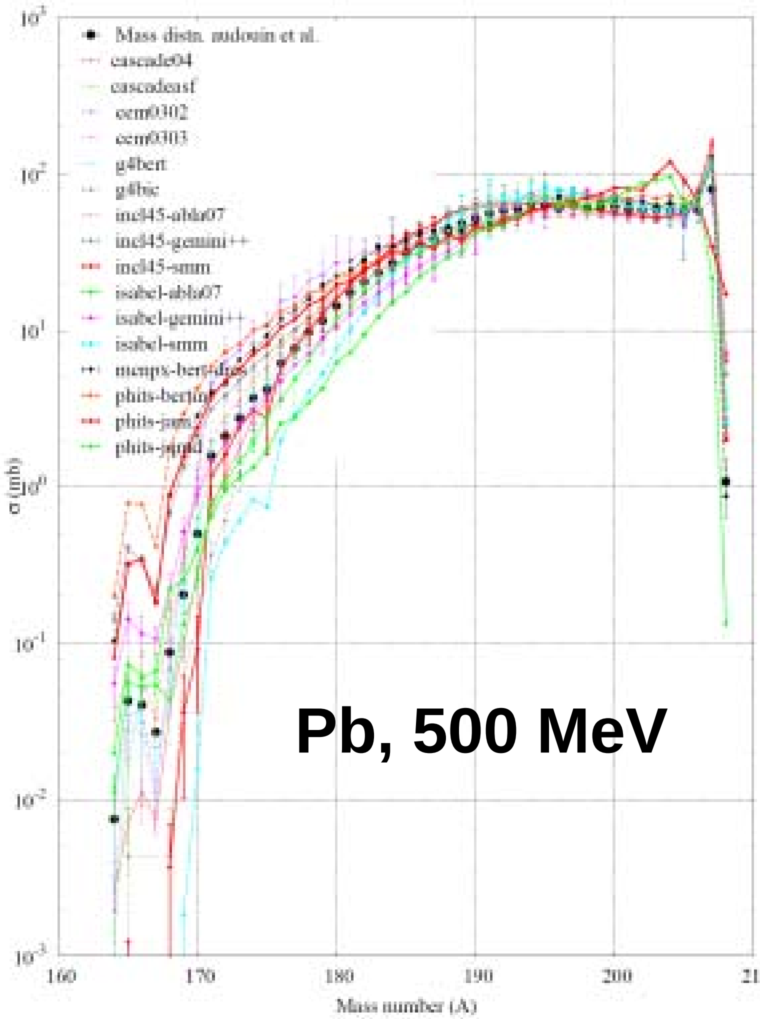



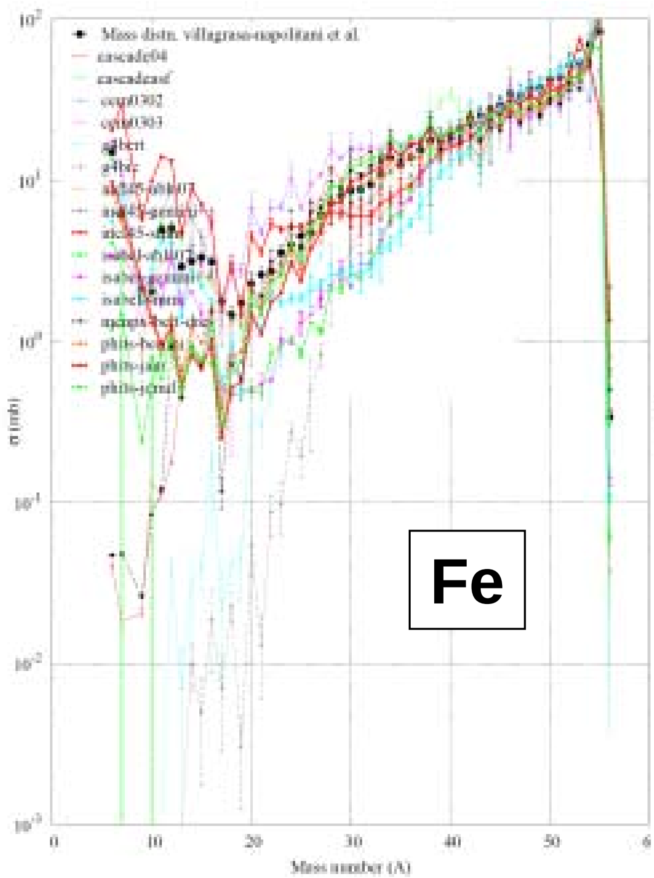



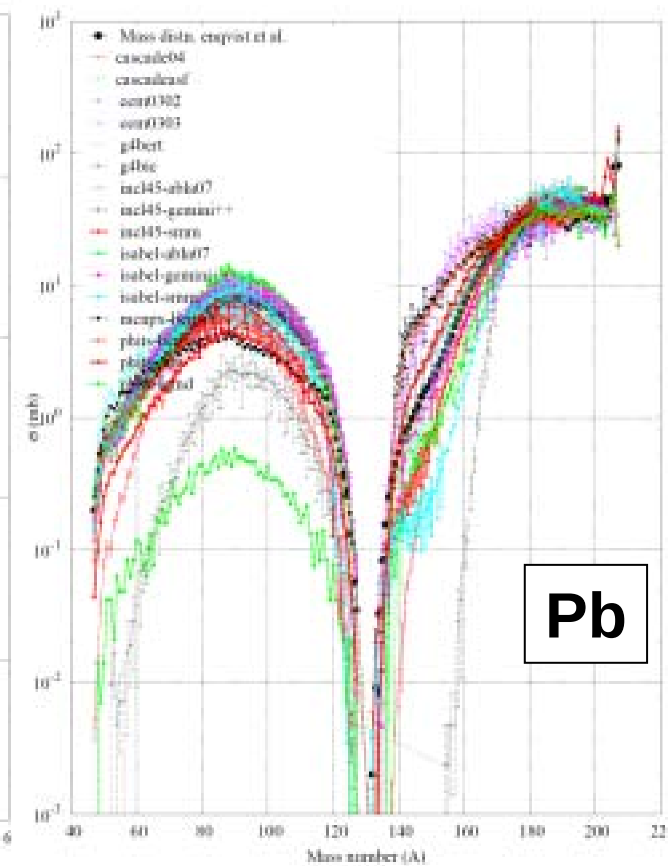
# Mass Distribution of Residues

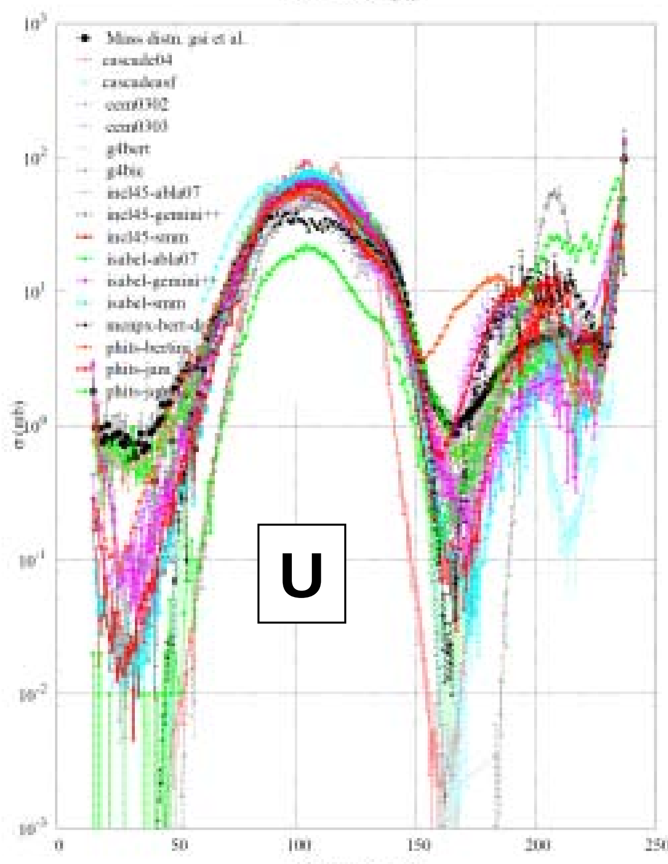


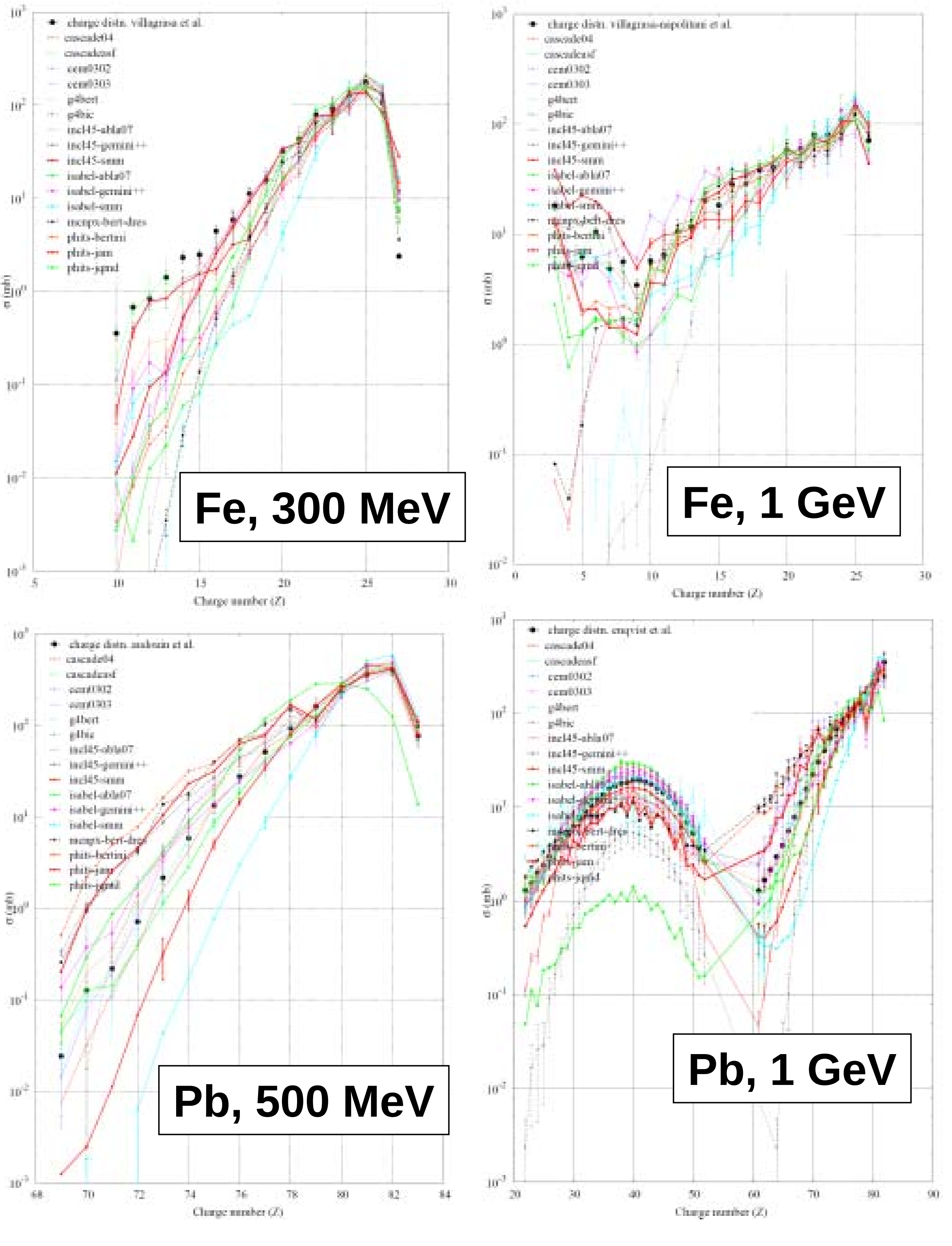
# Mass Distribution of Residues at 1 GeV



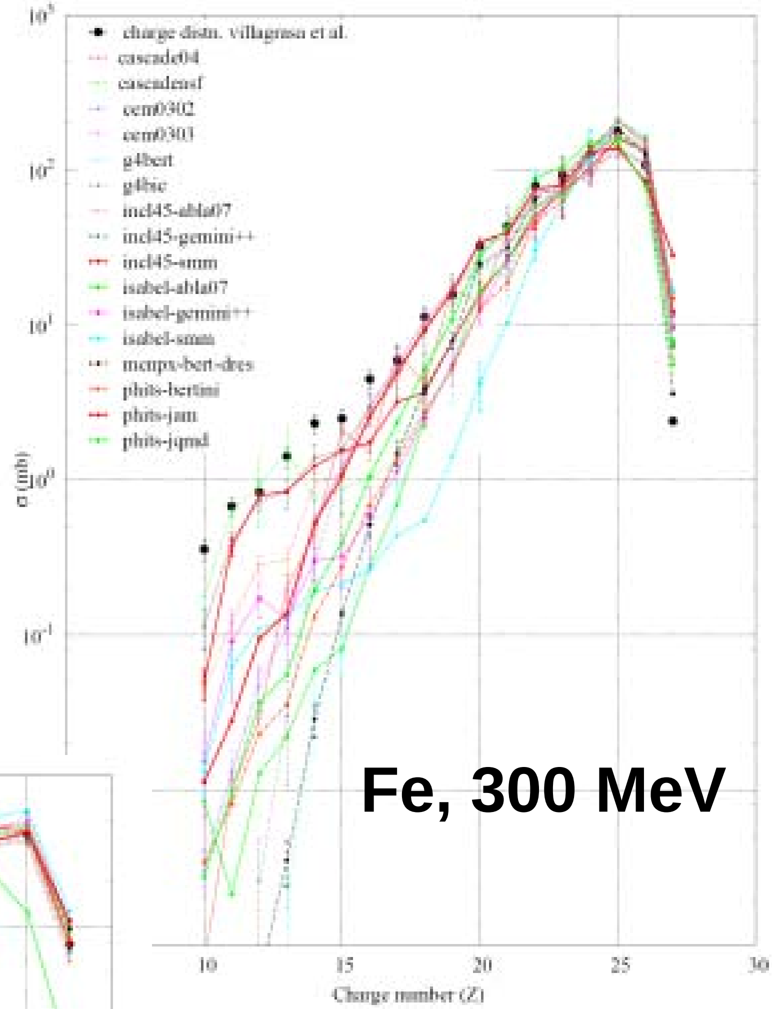



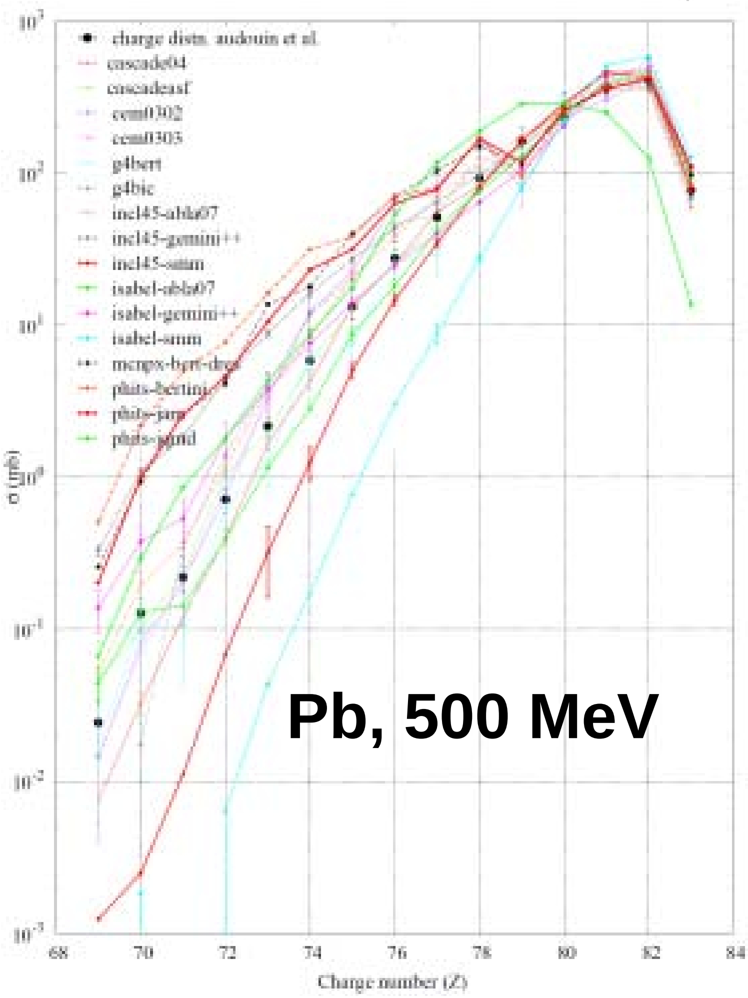



# Charge Distribution of Residues

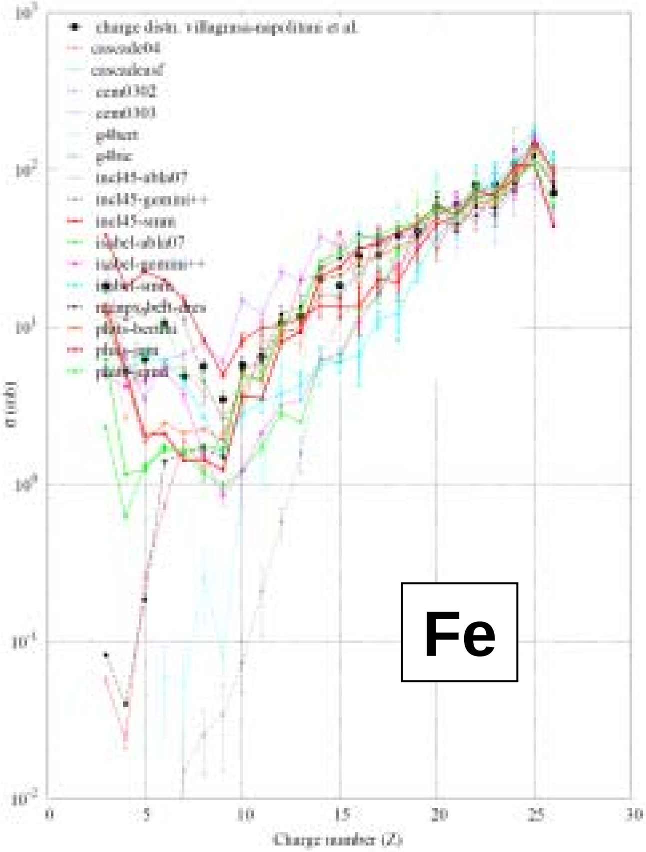


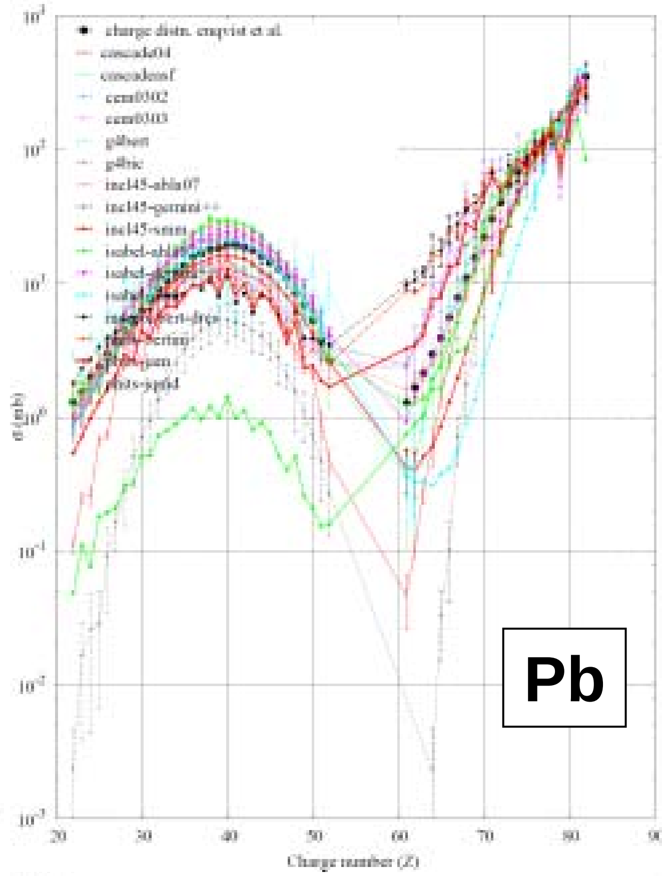
# Charge Distribution of Residues

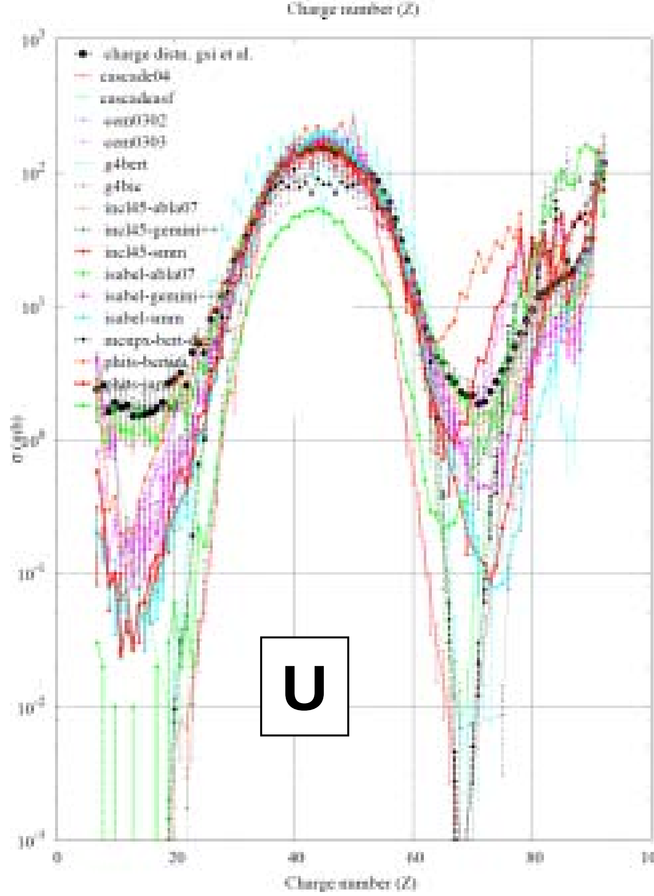


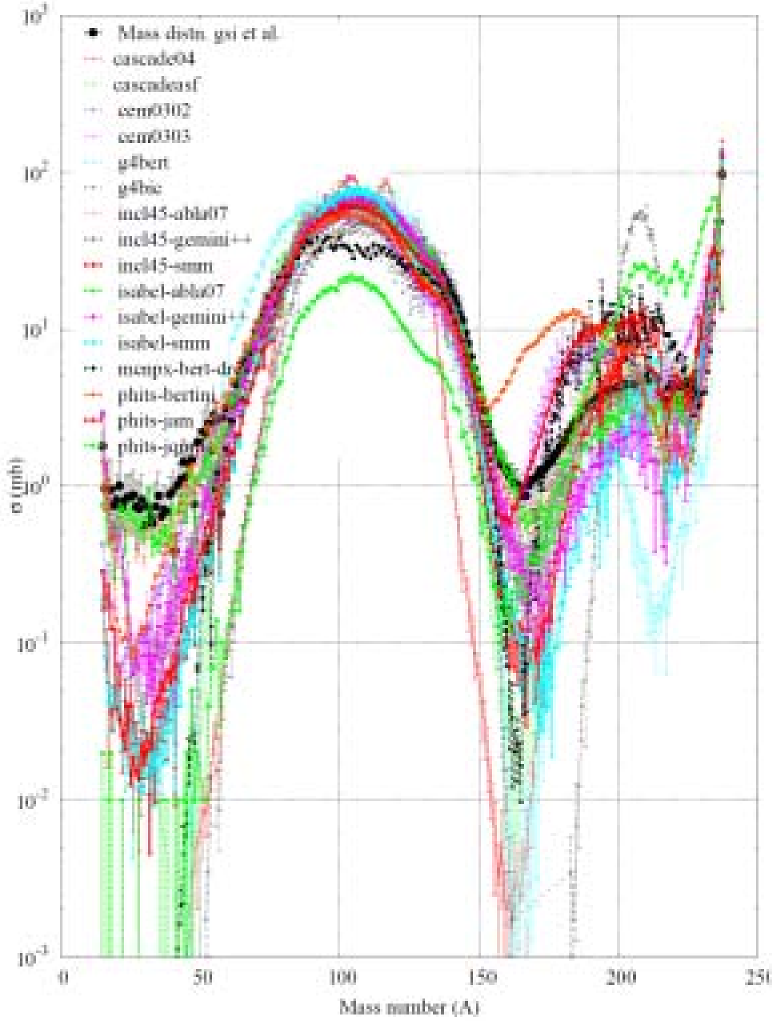
# Charge Distribution of Residues at 1 GeV

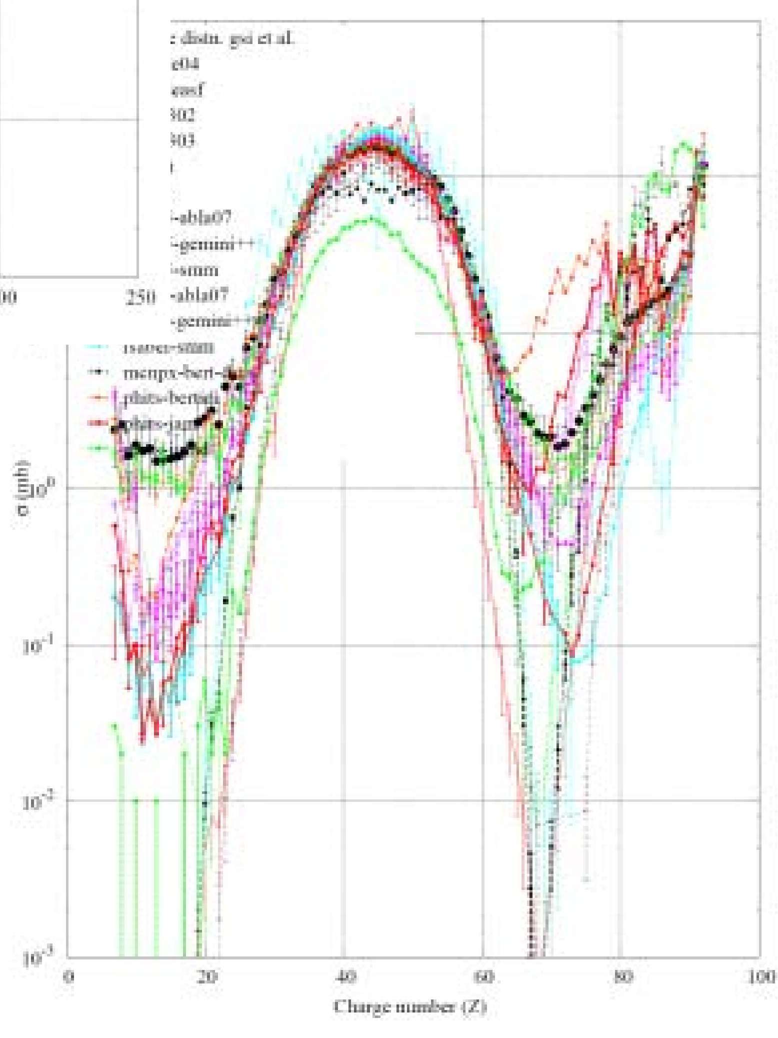


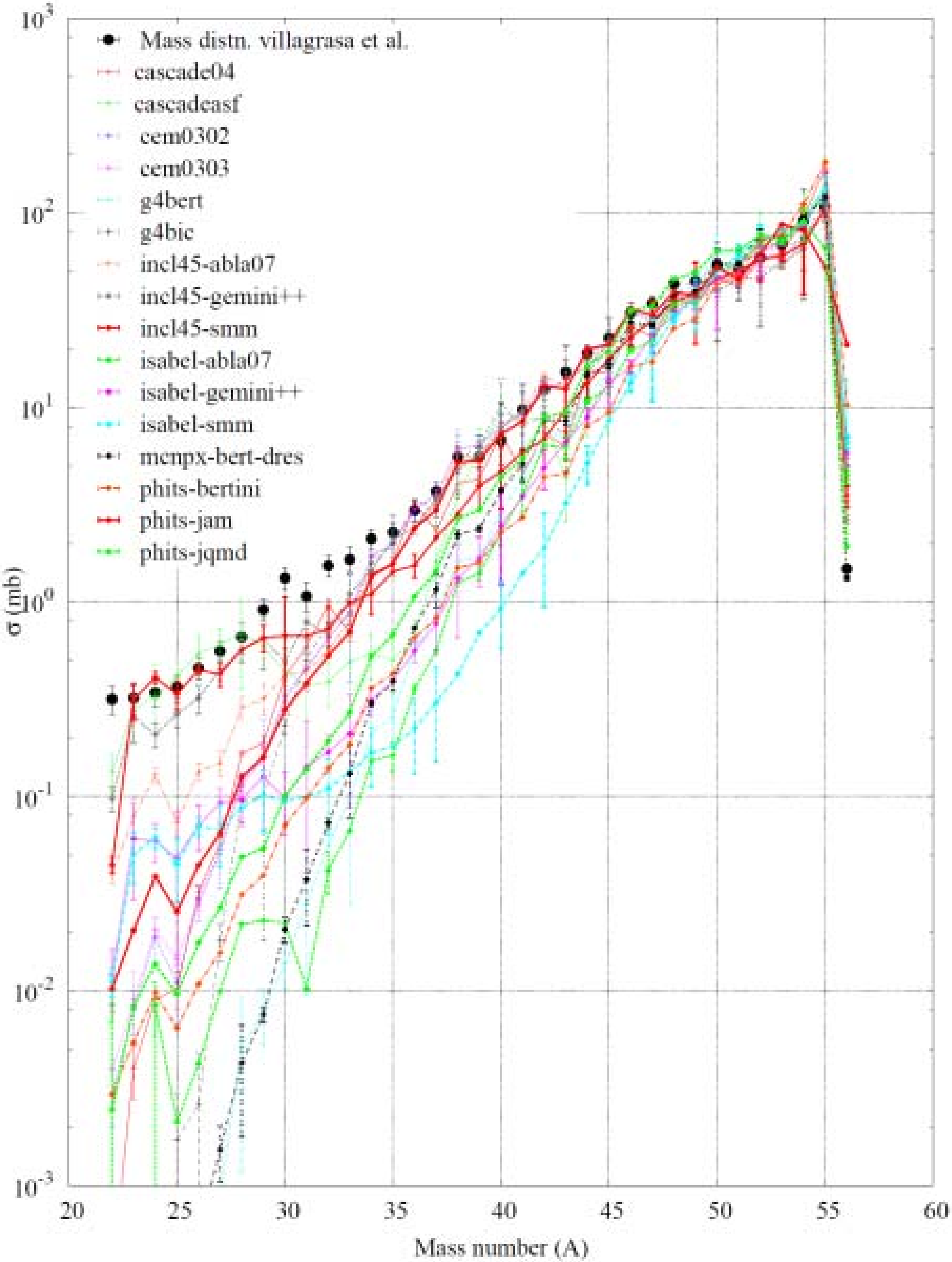
# Mass and Charge Distribution of Residues from Uranium at 1 GeV



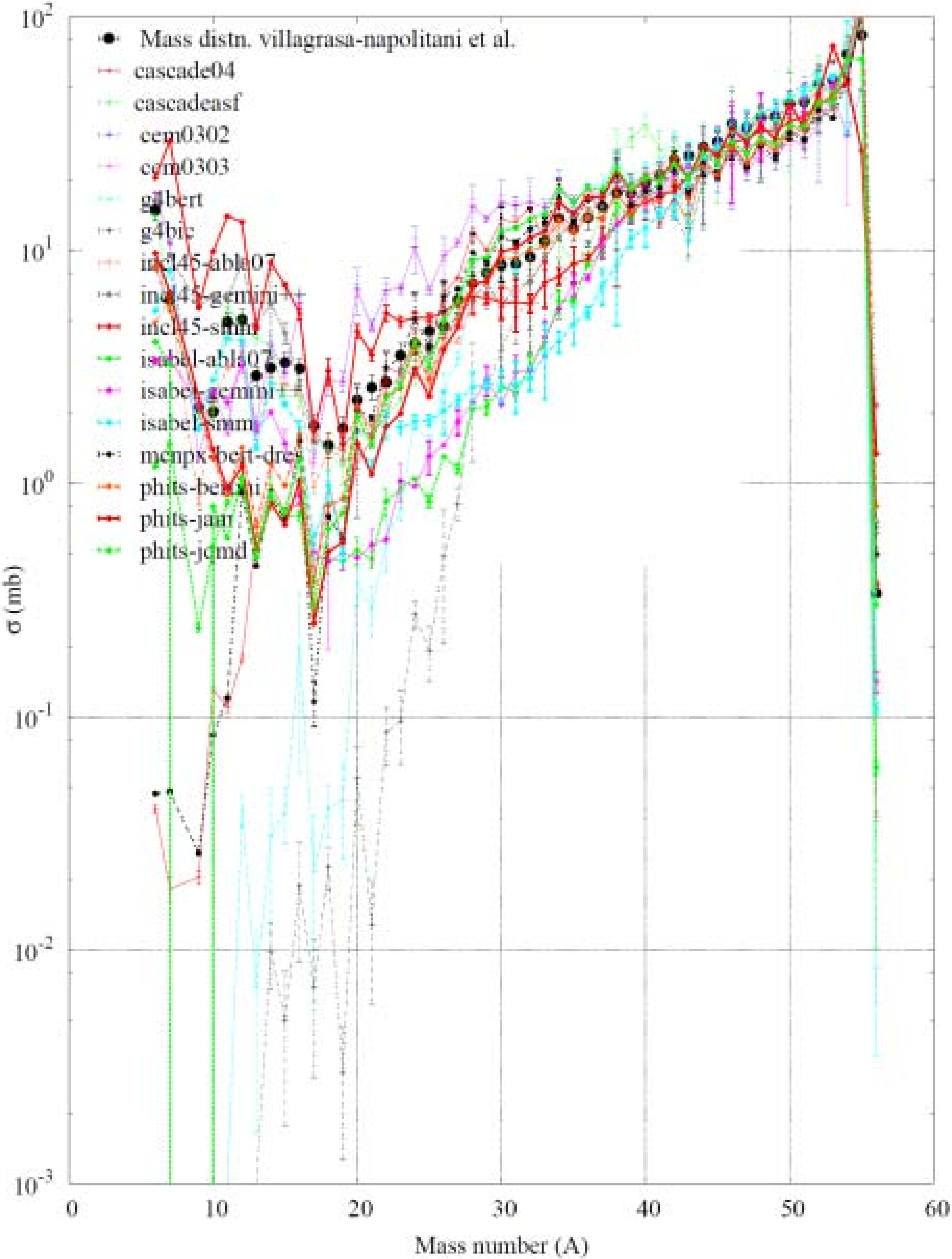



# Mass distributions

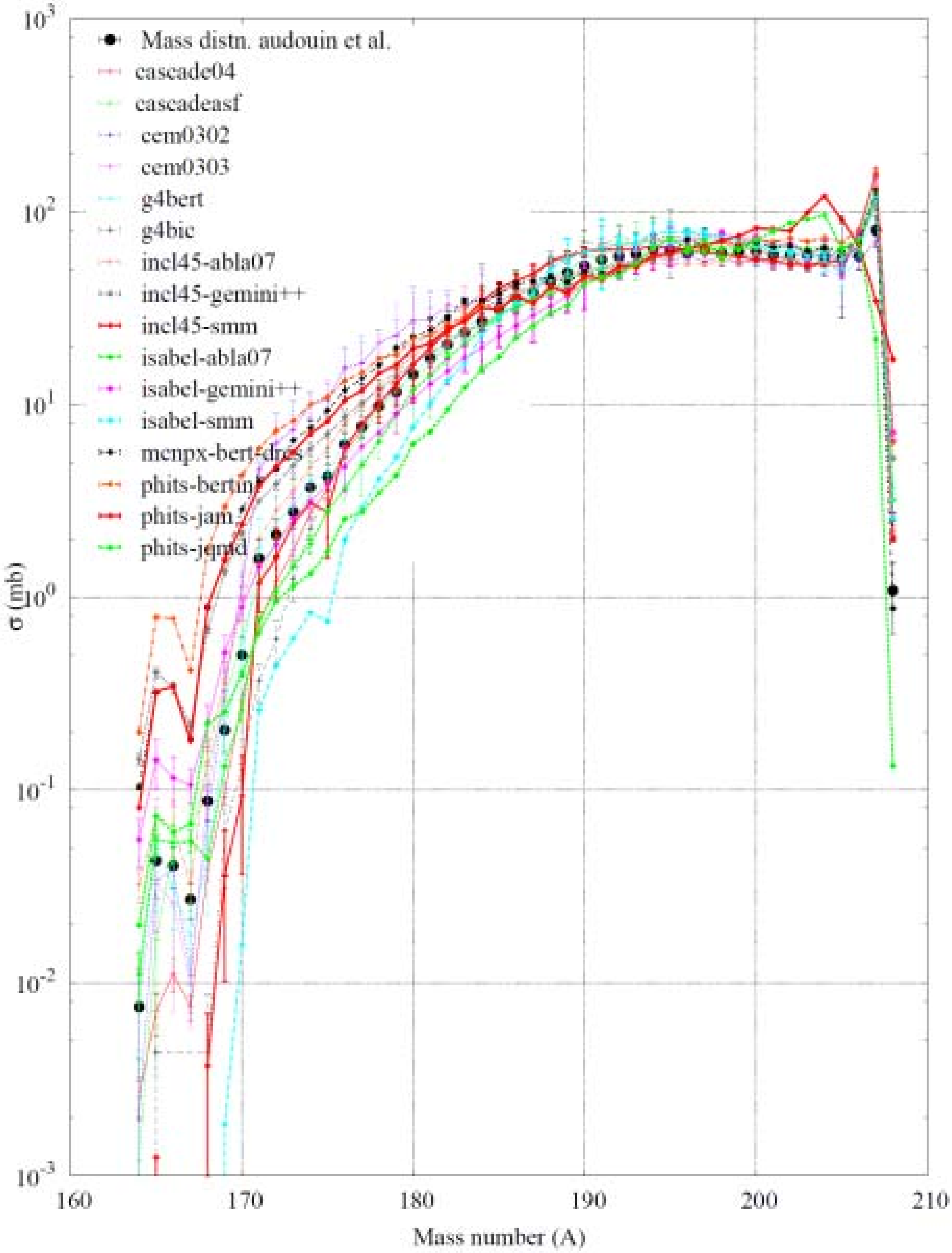
# Mass Distribution of Residues
# Iron @ 300 MeV



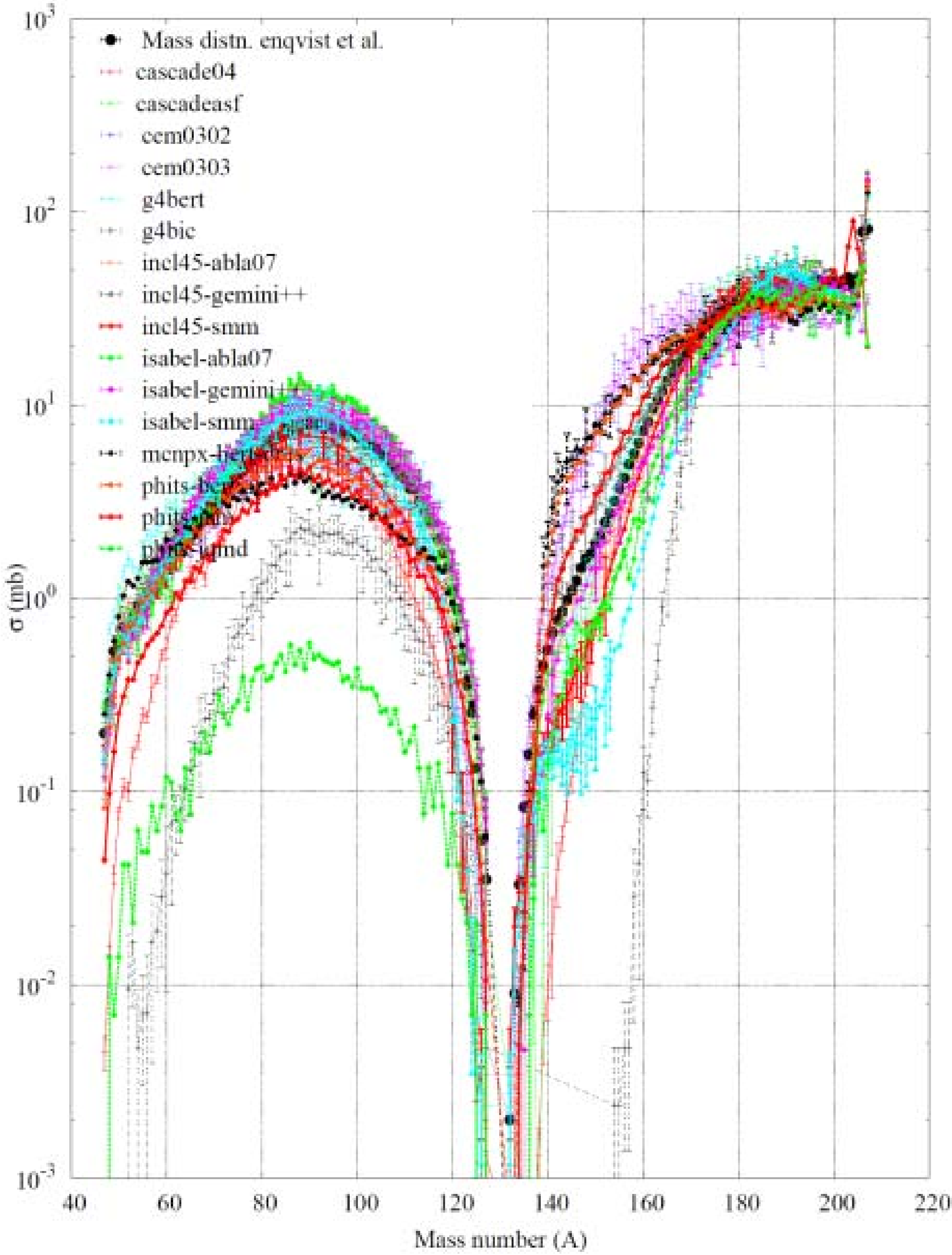
# Mass Distribution of Residues
# Iron @ 1 GeV



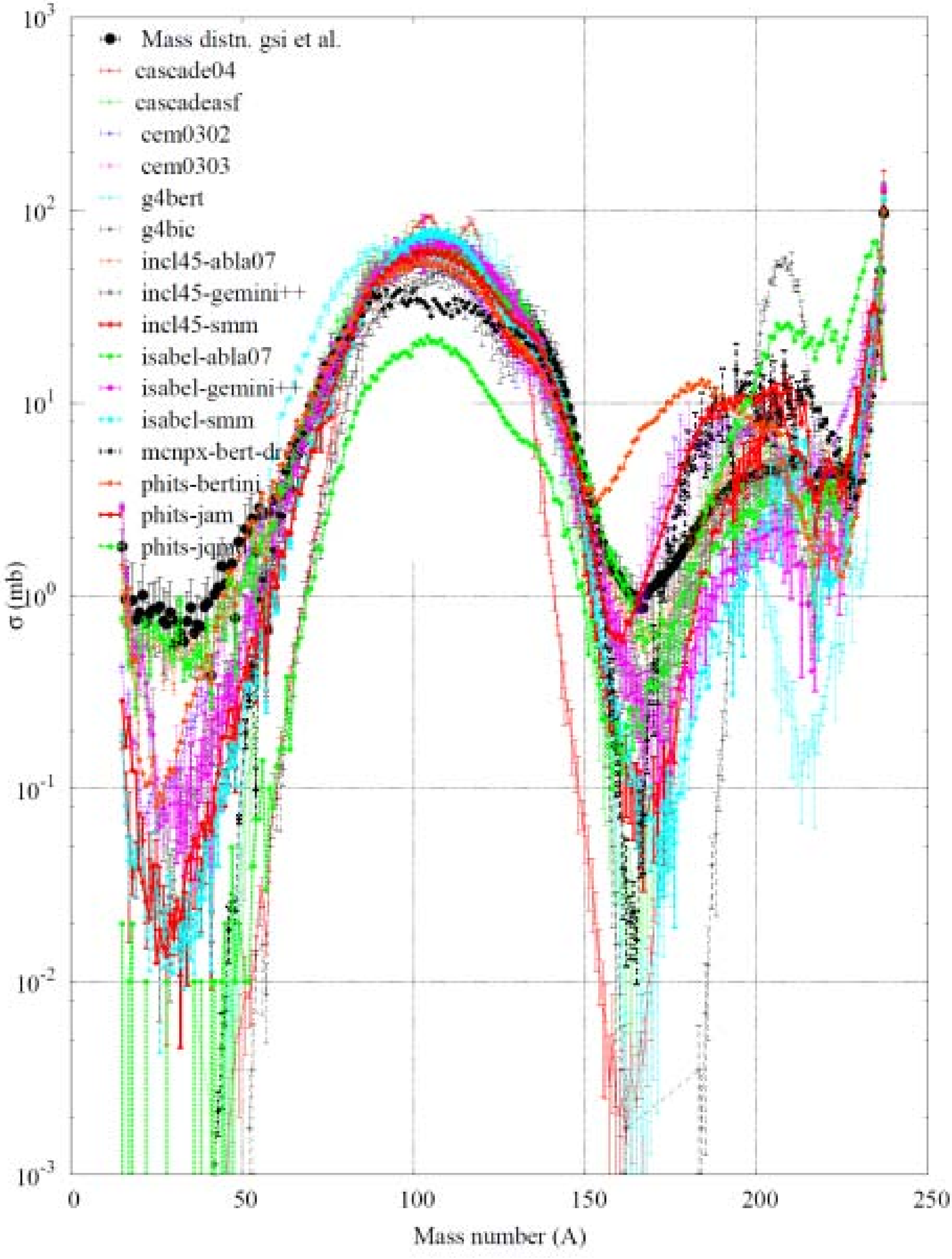
# Mass Distribution of Residues
# Lead @ 500 MeV



# Mass Distribution of Residues
# Lead @ 1 GeV

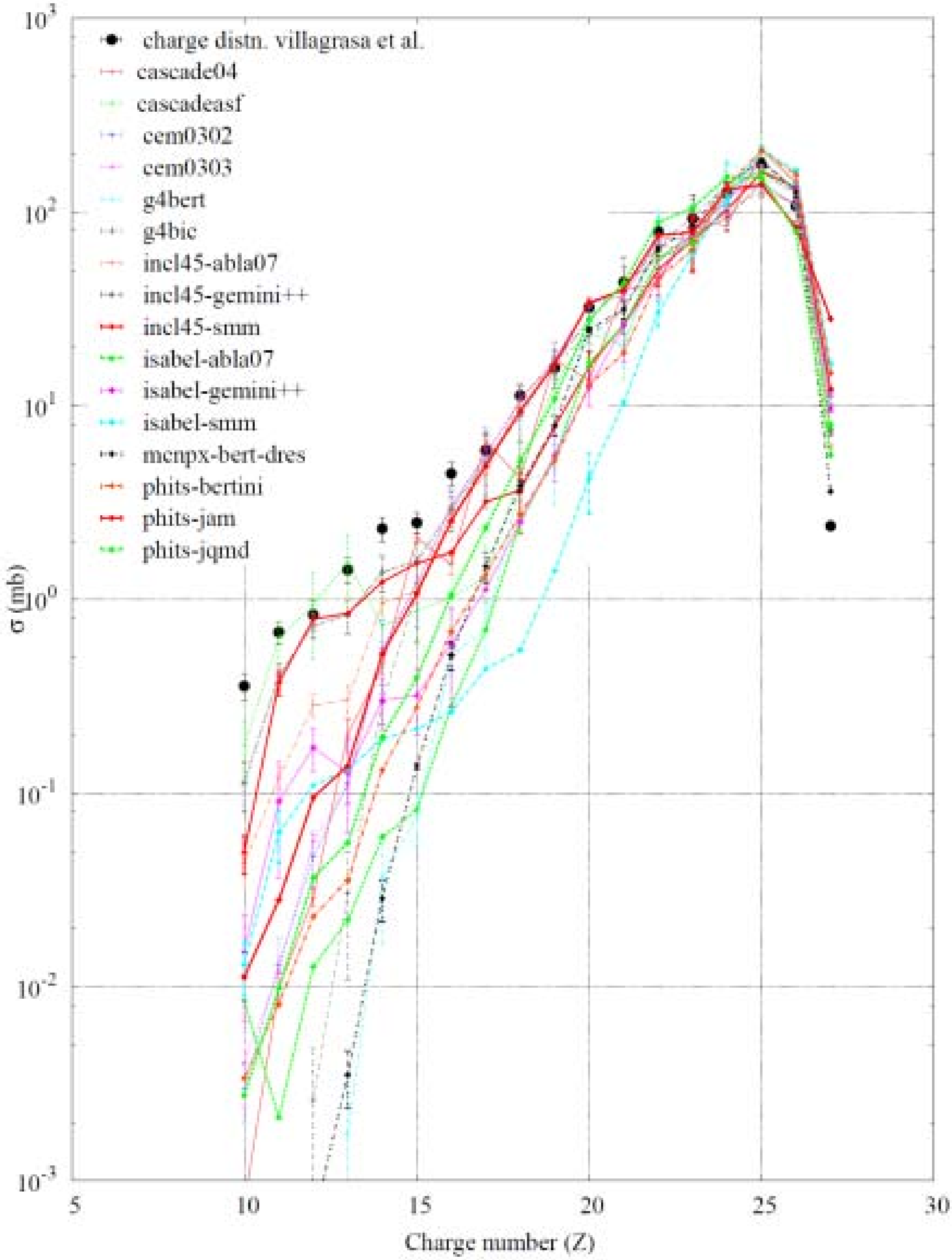


# Mass Distribution of Residues
# Uranium @ 1 GeV

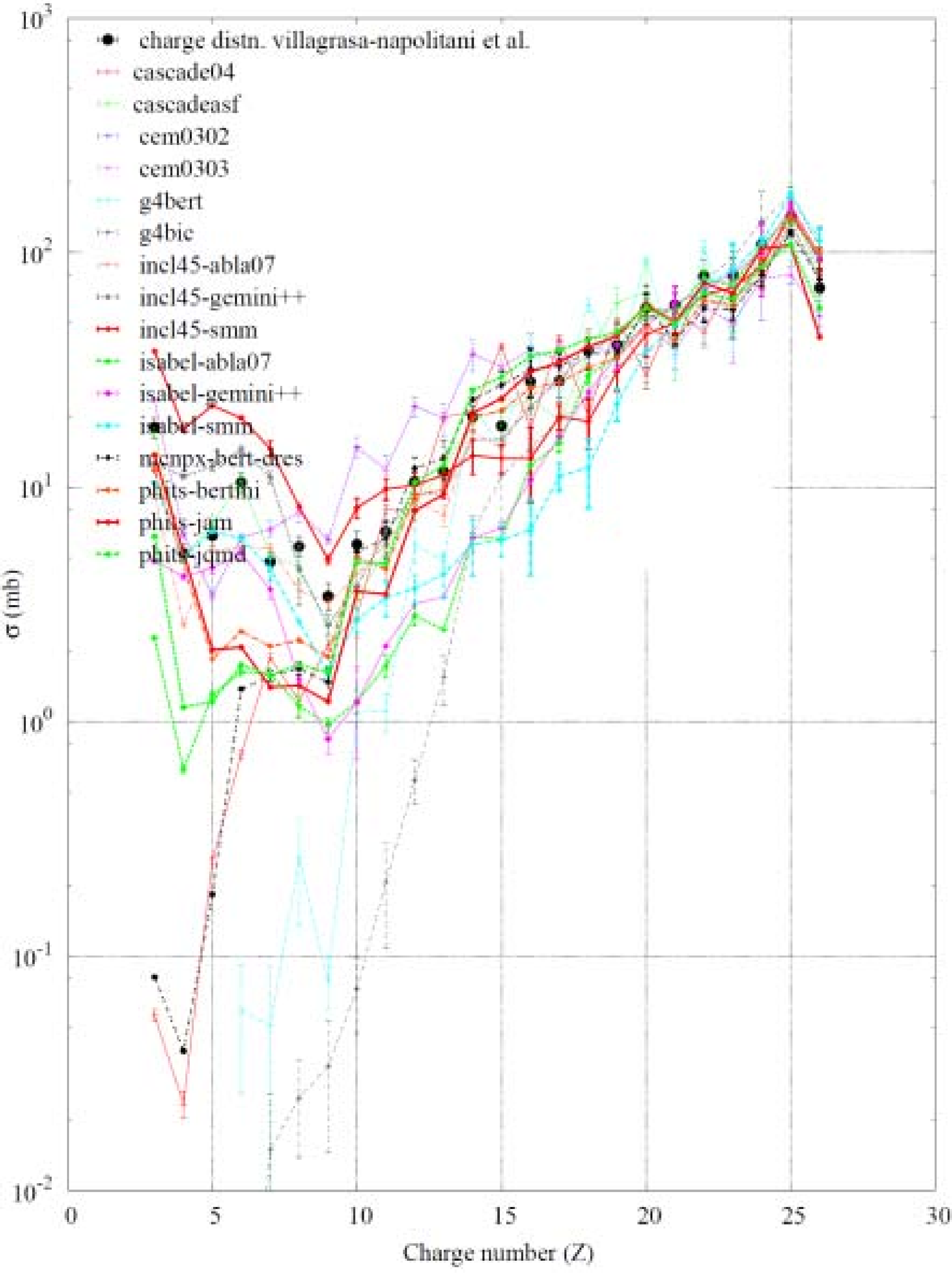


# Charge distributions

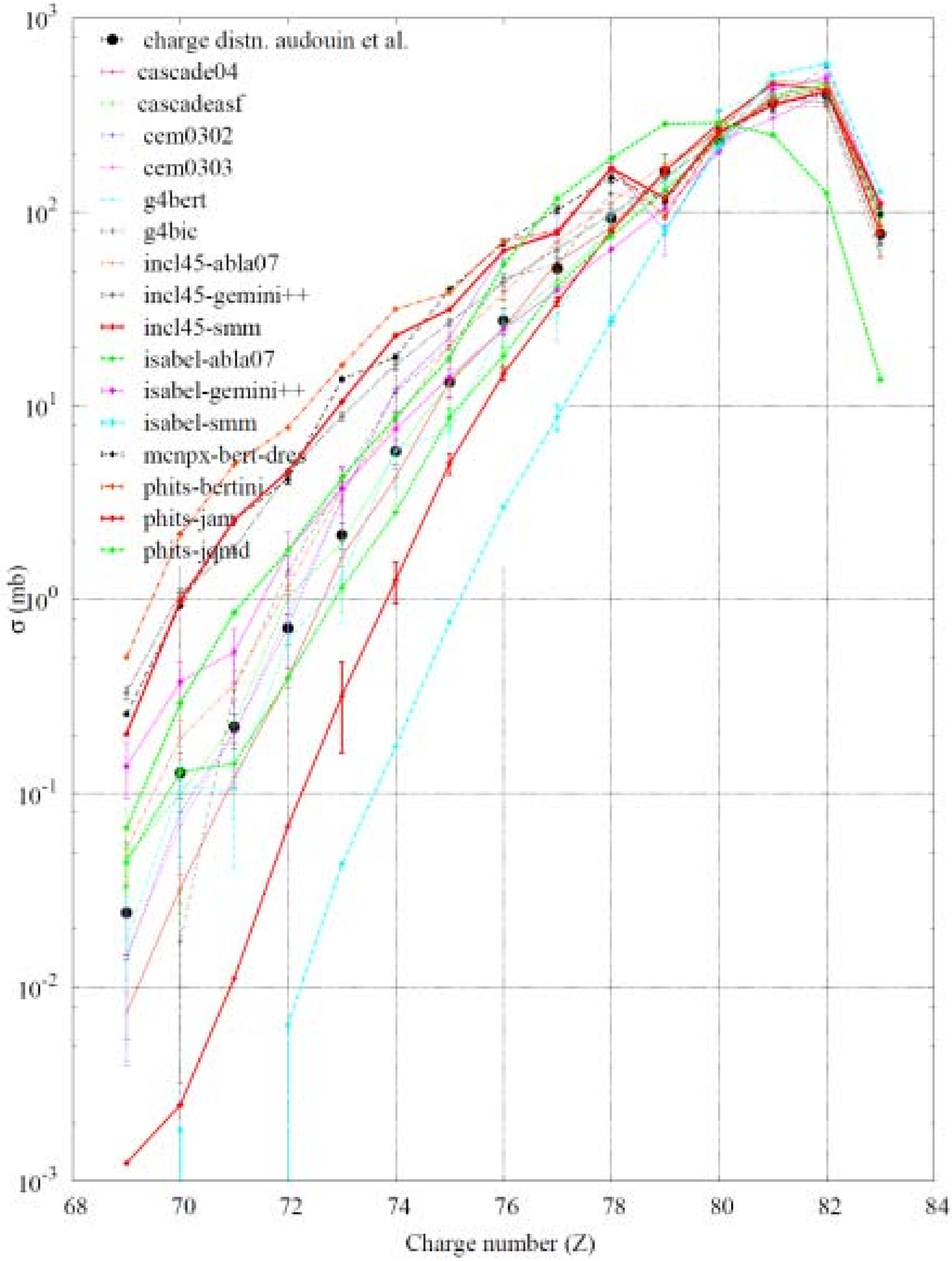
# Charge distribution
# Iron @ 300 MeV



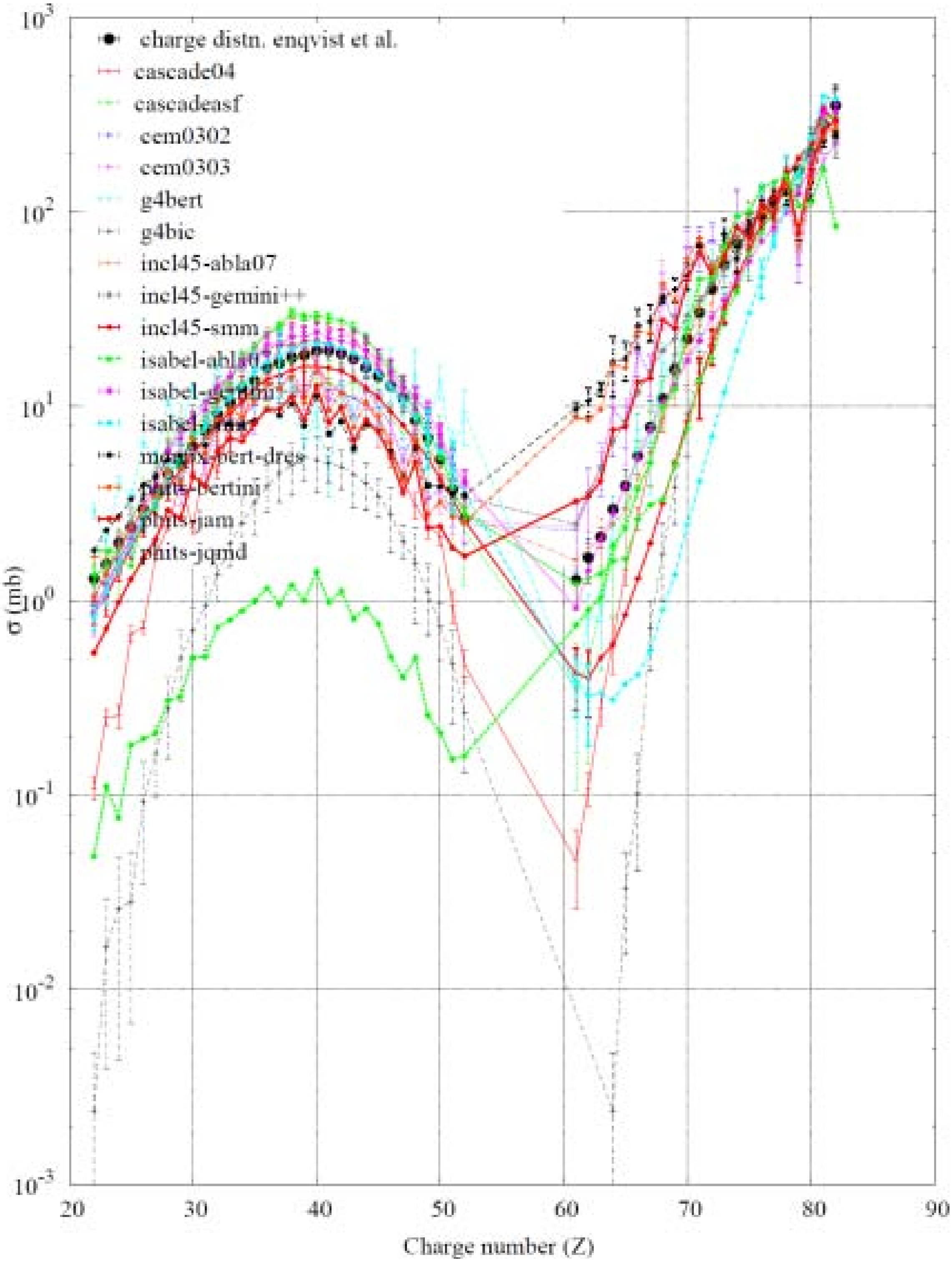
# Charge distribution
# Iron @ 1 GeV



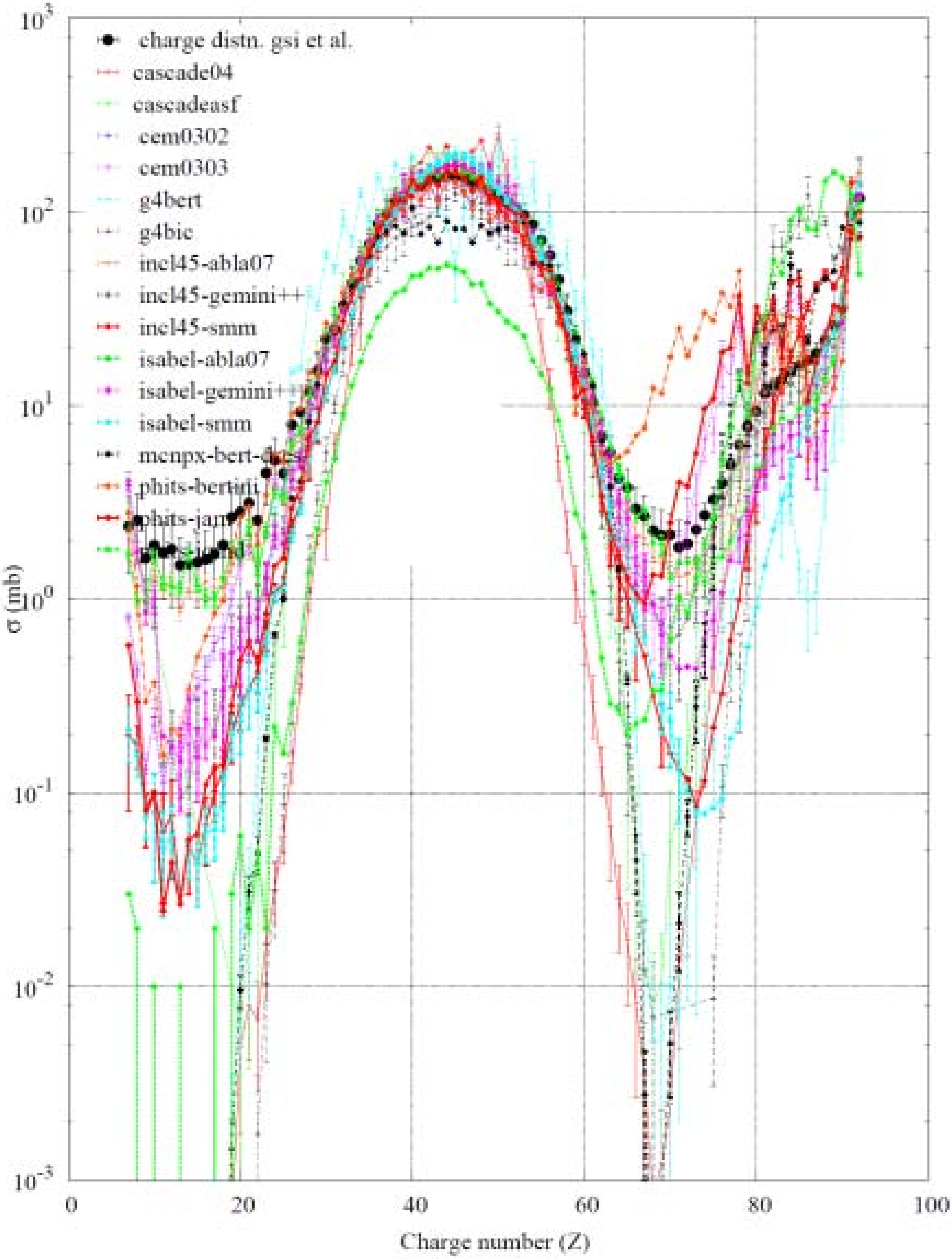
# Charge distribution
# Lead @ 500 MeV



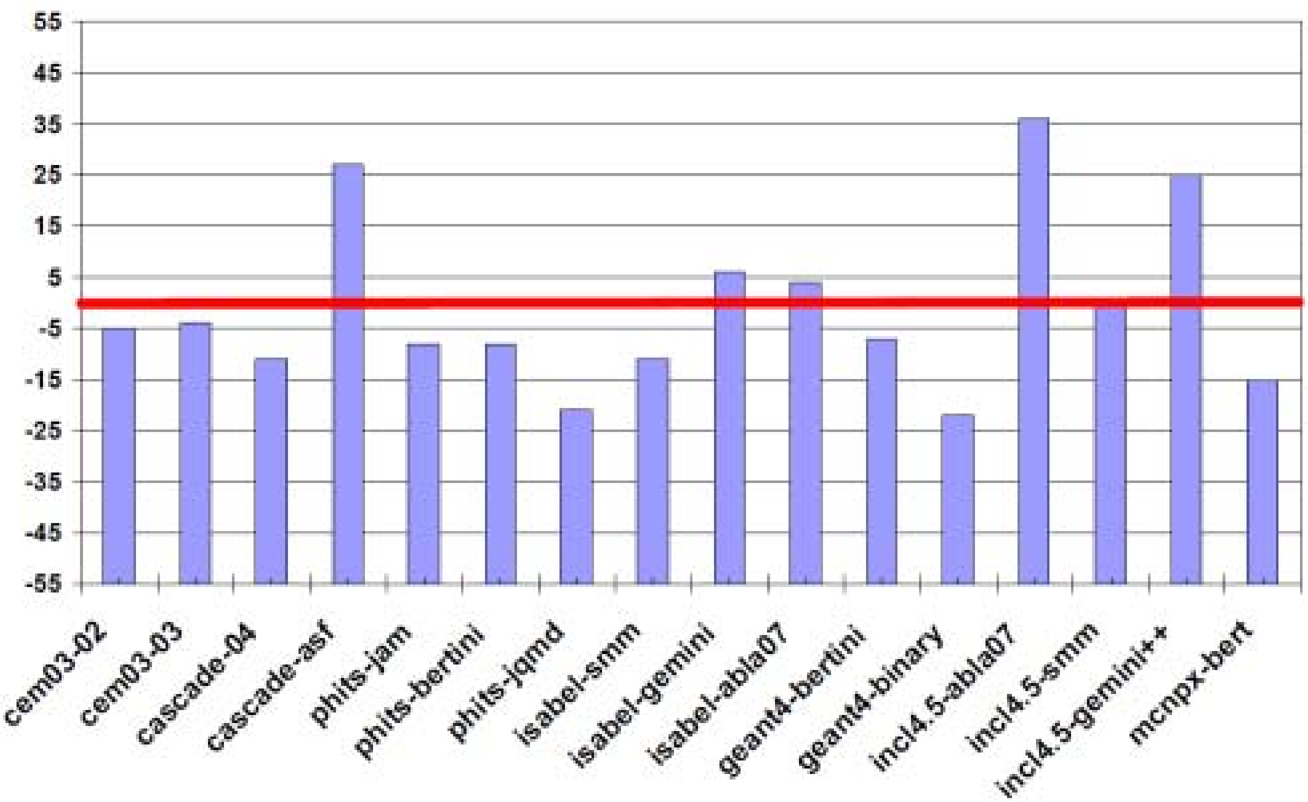
# Charge distribution
# Lead @ 1 GeV



# Charge distribution
# Uranium @ 1 GeV



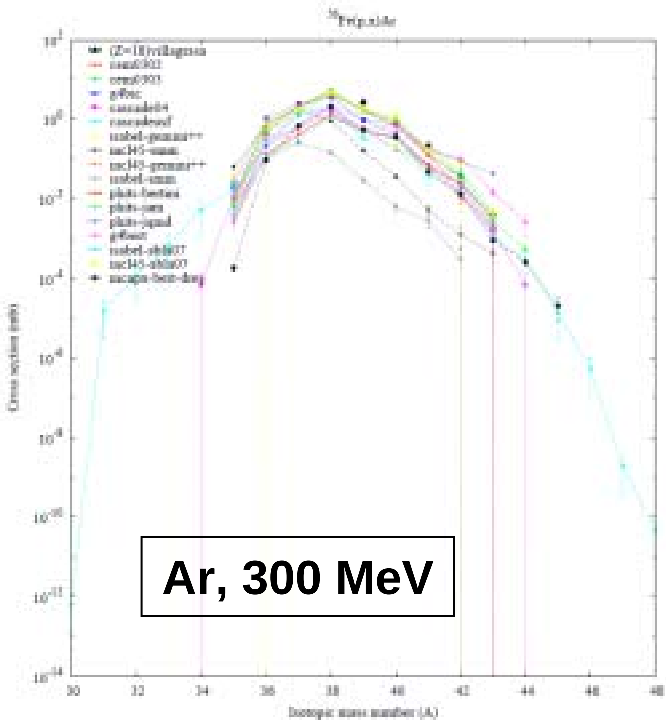
# Rating of the results of 15 participants for predicting the mass and charge distributions measured by inverse kinematics for iron, lead, and uranium at all energies

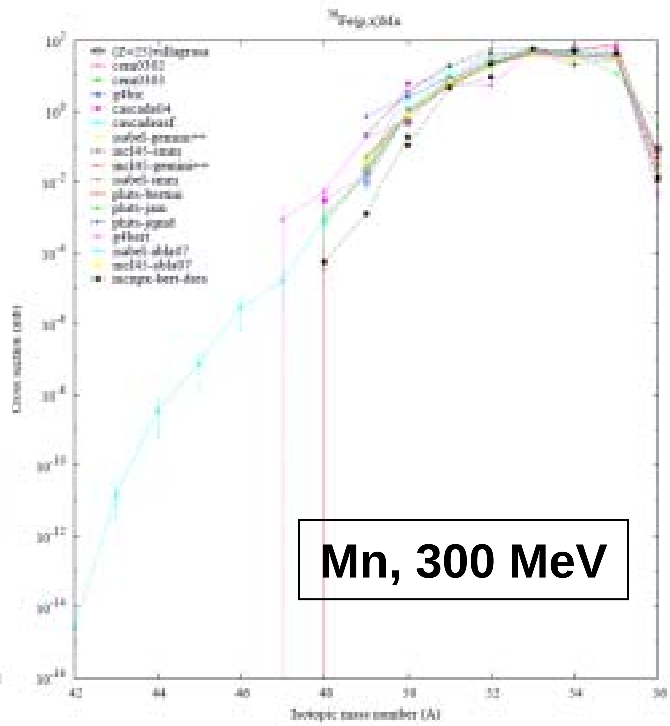


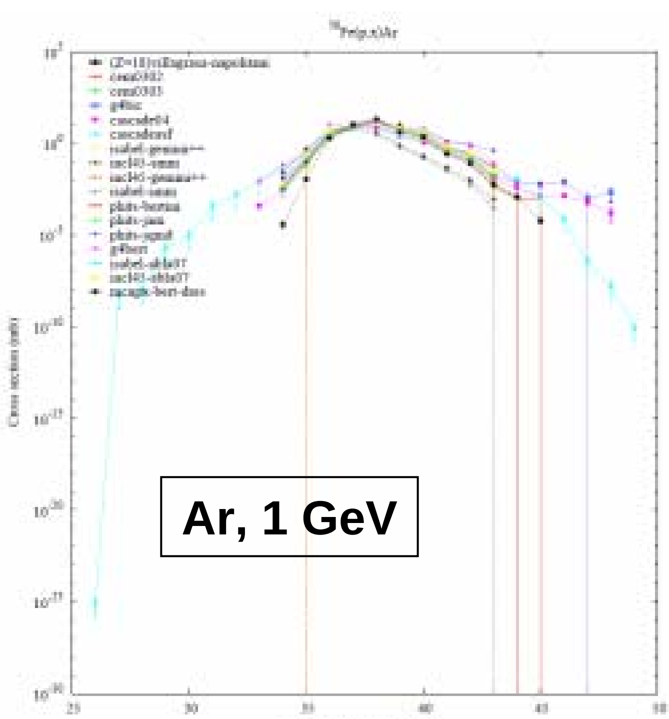

**Possible maximum: 56 points**

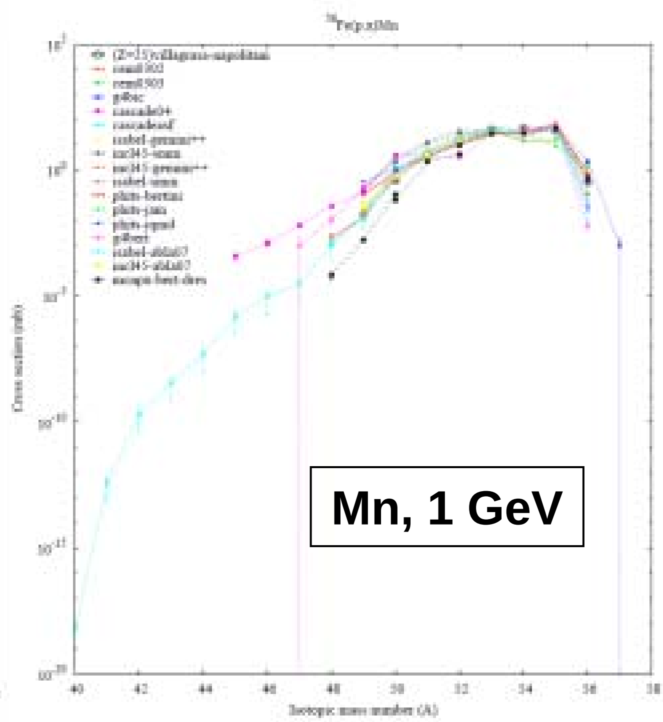
# Isotope distributions

# Exemplary isotope distributions for spallation products and target near products; iron 300 MeV and 1 GeV

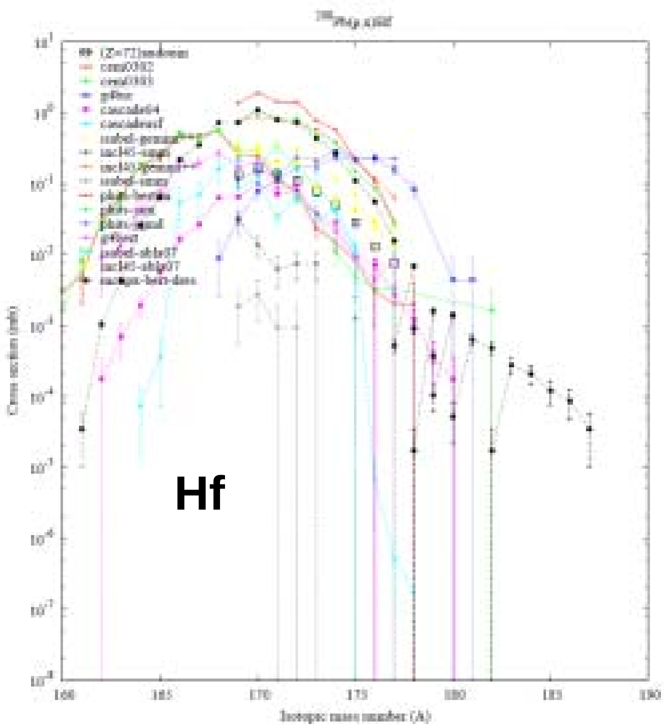


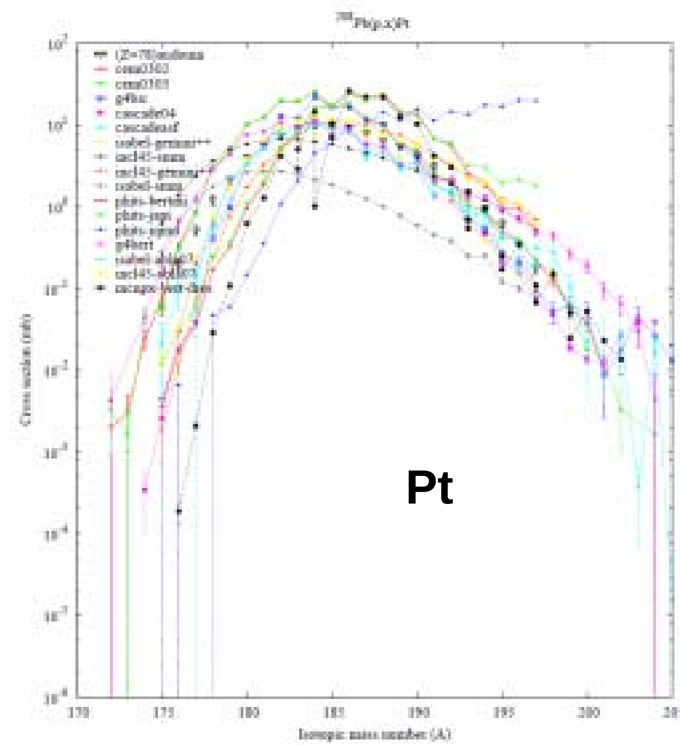



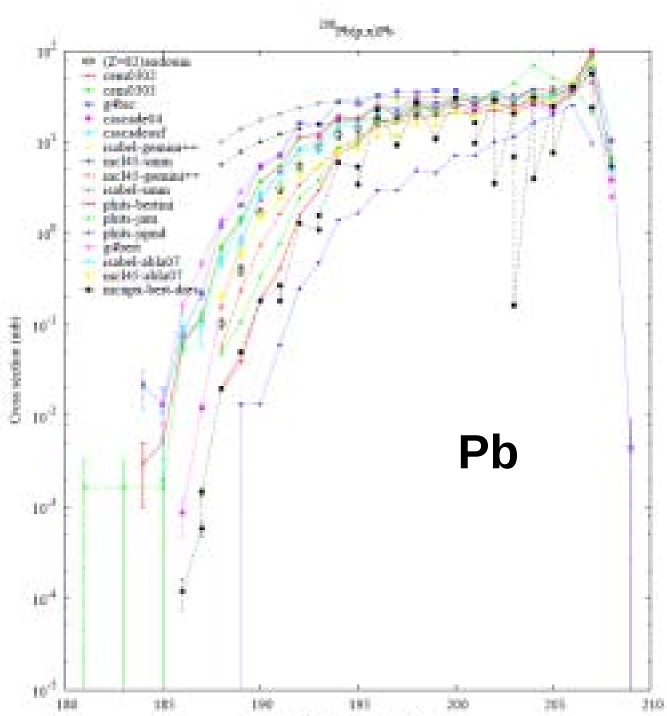



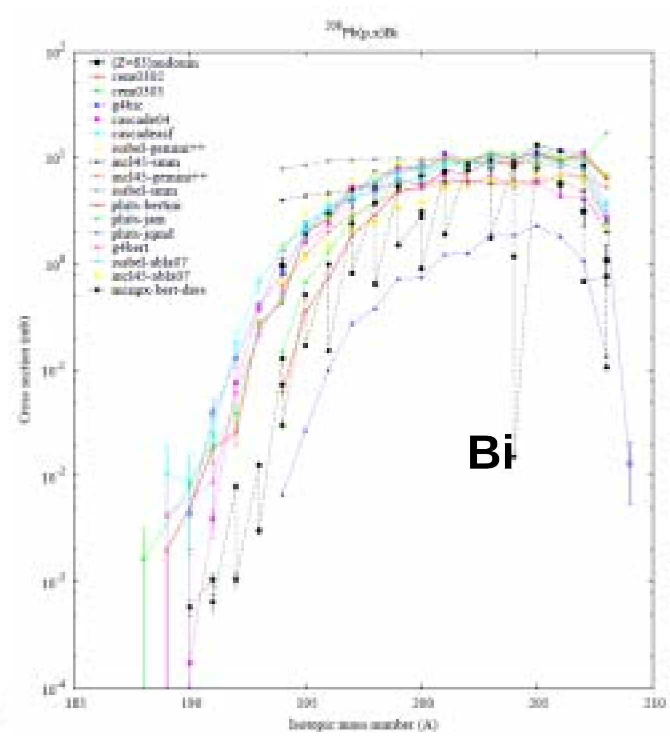



# Exemplary isotope distributions for spallation products and target near products; lead 500 MeV

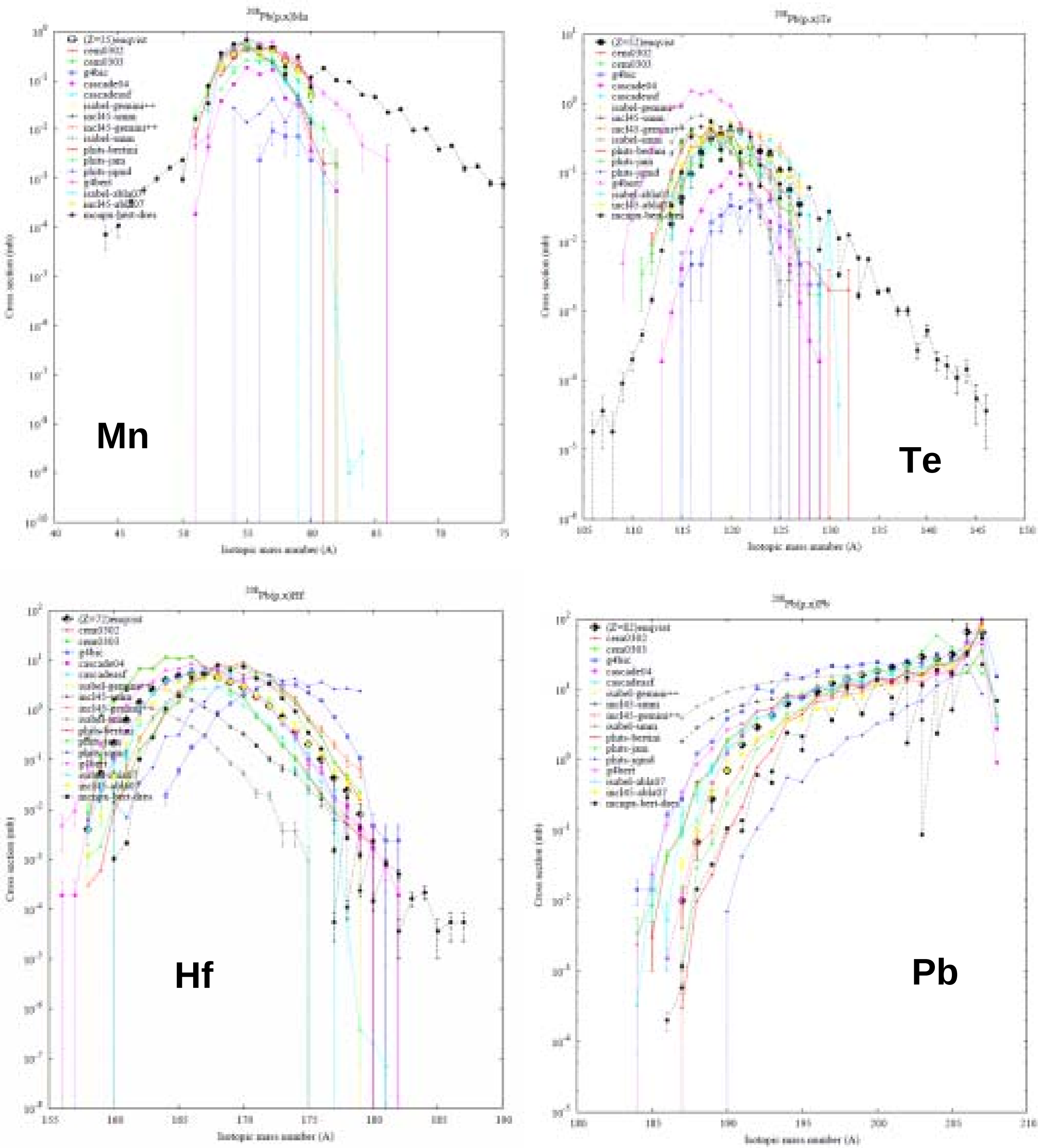


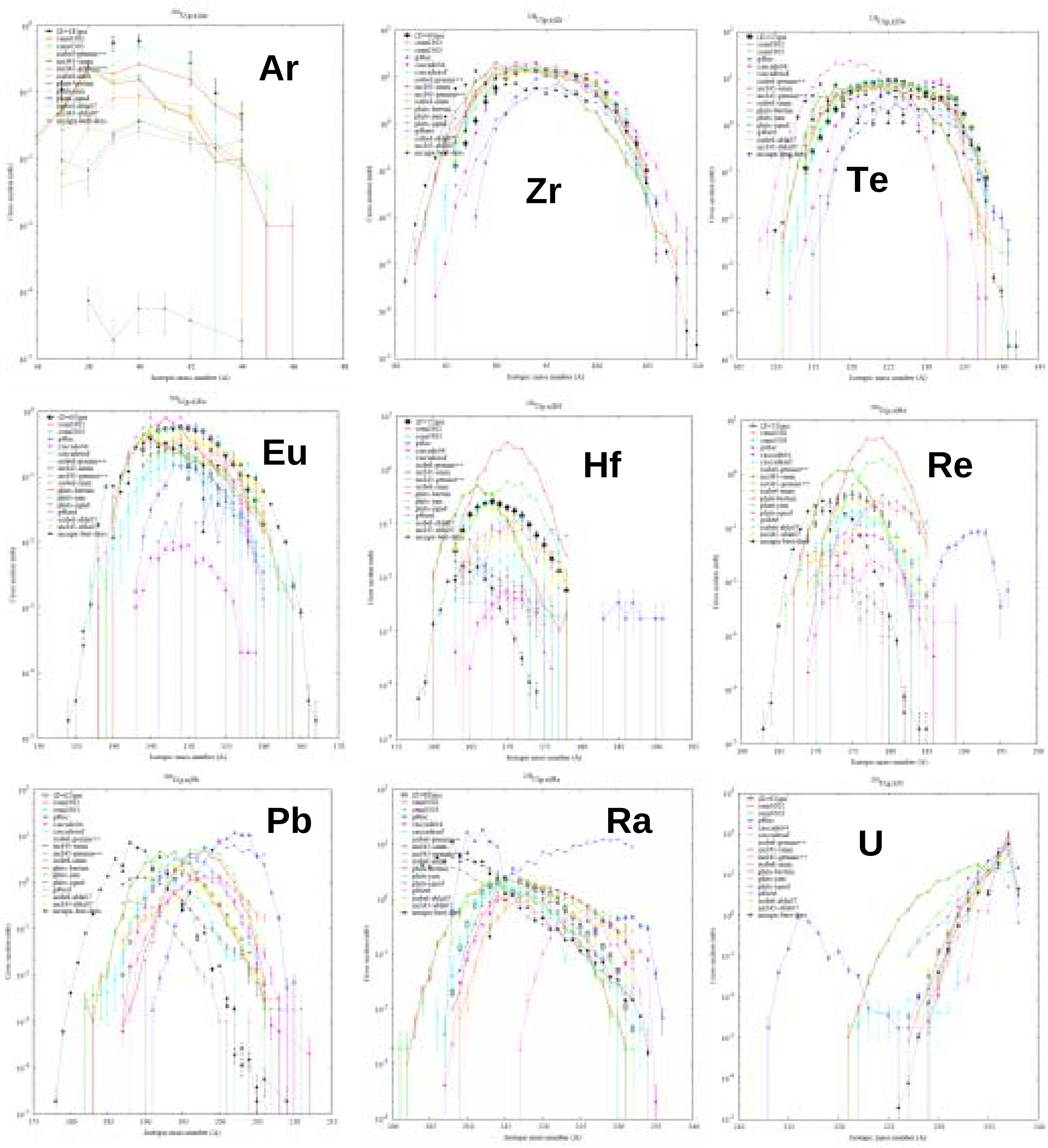



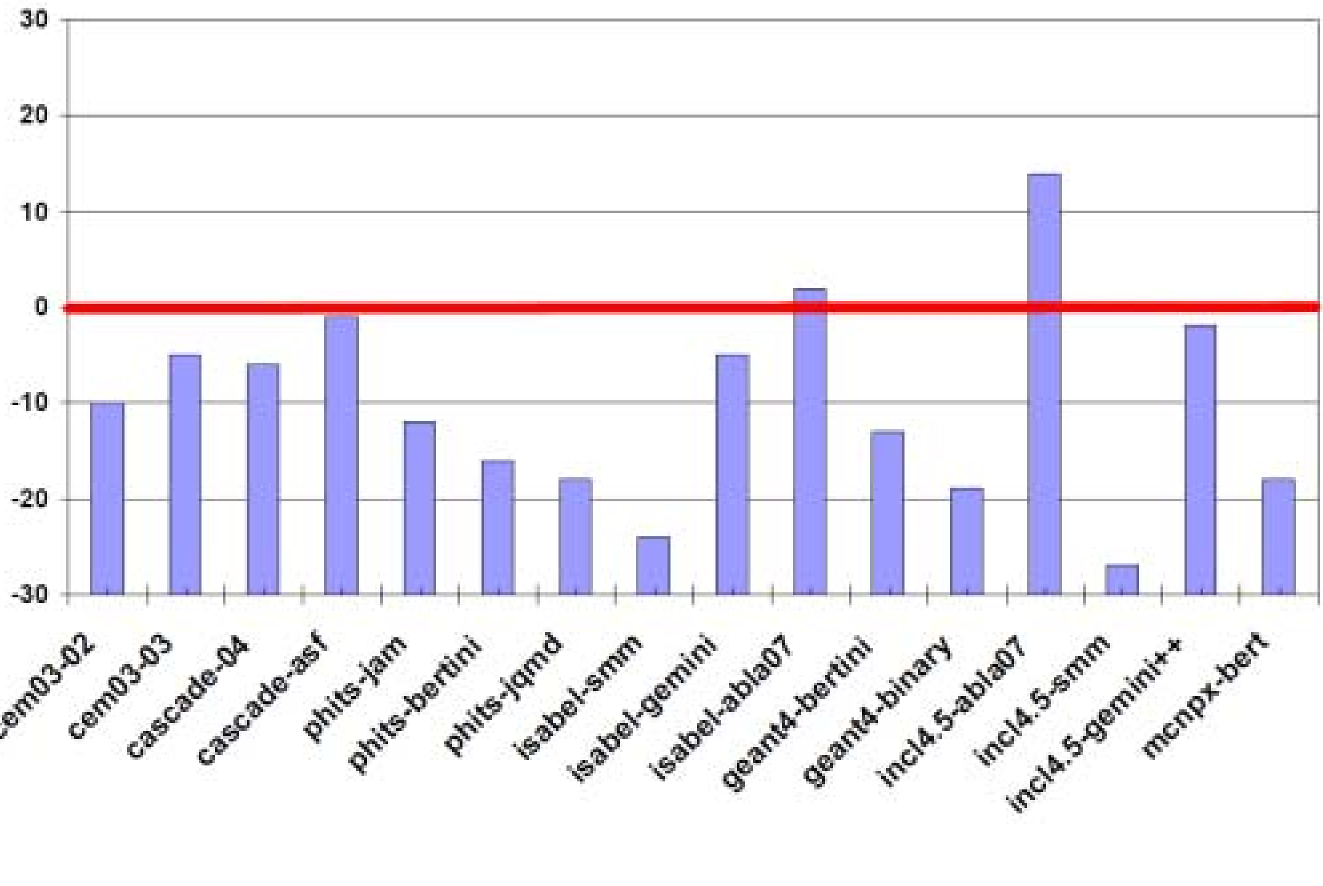
# Exemplary isotope distributions for light products, fission products, spallation products, and target near products; lead at 1 GeV



# Exemplary isotope distributions for light products, fission products, spallation products, and target near products; uranium 1 GeV

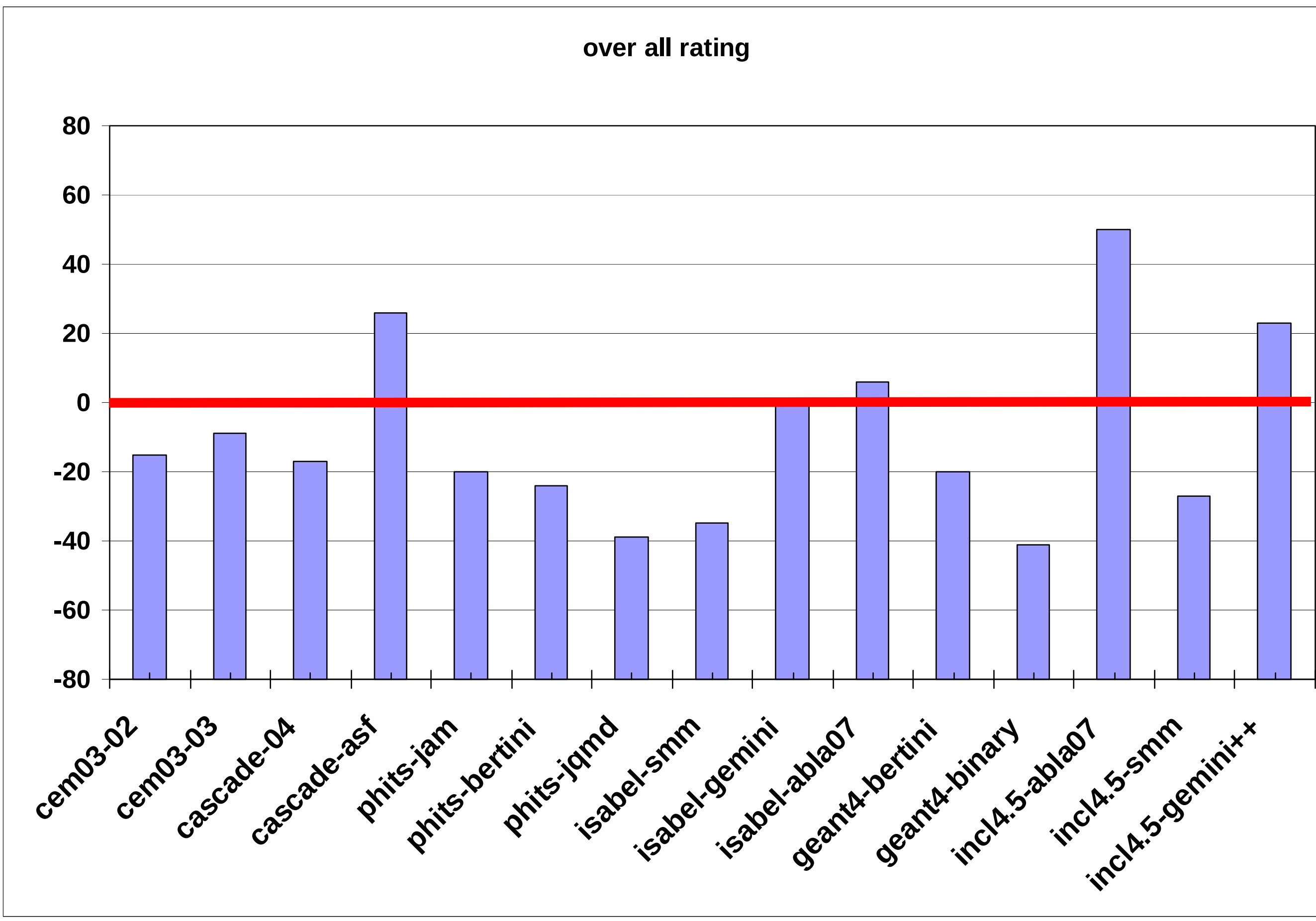


# Rating of the results of 15 participants for predicting the isotope distributions measured by inverse kinematics for iron, lead, and uranium at all energies.




**Possible maximum: 28 points**

# Rating of the results of 15 participants for predicting the mass, charge, and isotope distributions measured by inverse kinematics for iron, lead, and uranium at all energies.




**Possible maximum: 84 points**

# Excitation Functions

For excitation functions the simple rating system cannot be applied. The energy dependencies have to be looked at in detail.

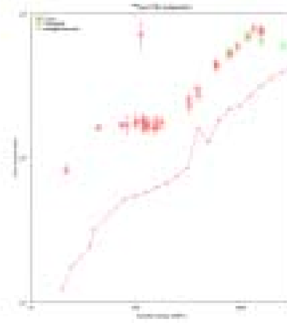
The Benchmark on Spallation Models provides a tool for continuous improvement of models and codes and for iterative comparisons.

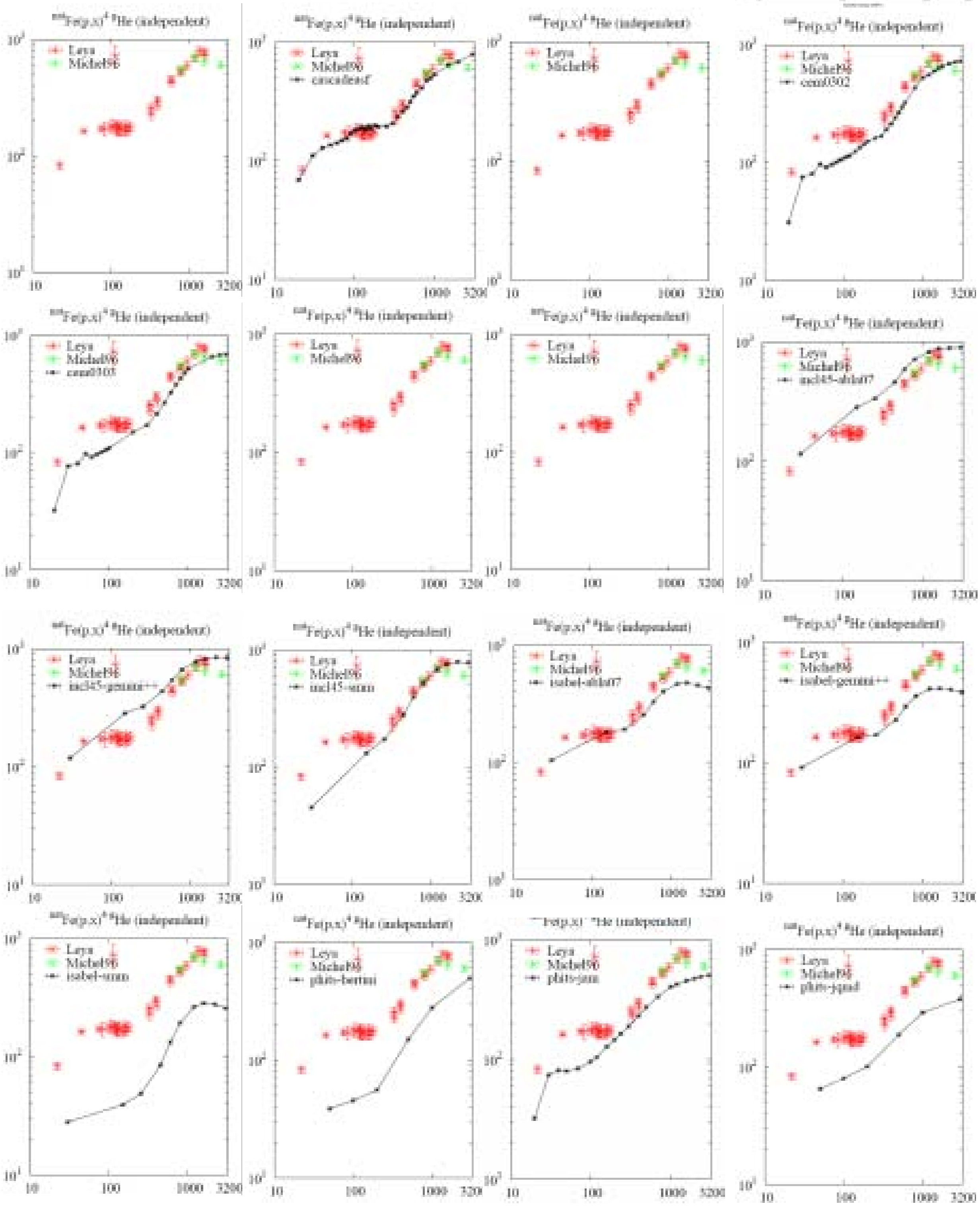
# Excitation Functions
# Order of Presentation

| | | | |
|---|---|---|---|
| | | | mcnpx-bert |
| cascade04 | cascadeasf | cascadex | cem0302 |
| cem0303 | g4bert | g4bic | incl45-abla07 |
| incl45-gemini++ | incl45-smm | isabel-abla07 | isabel-gemini++ |
| isabel-smm | phits-bertini | phits-jam | phits-jqmd |

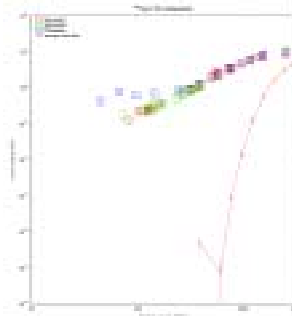
# Excitation Functions Iron

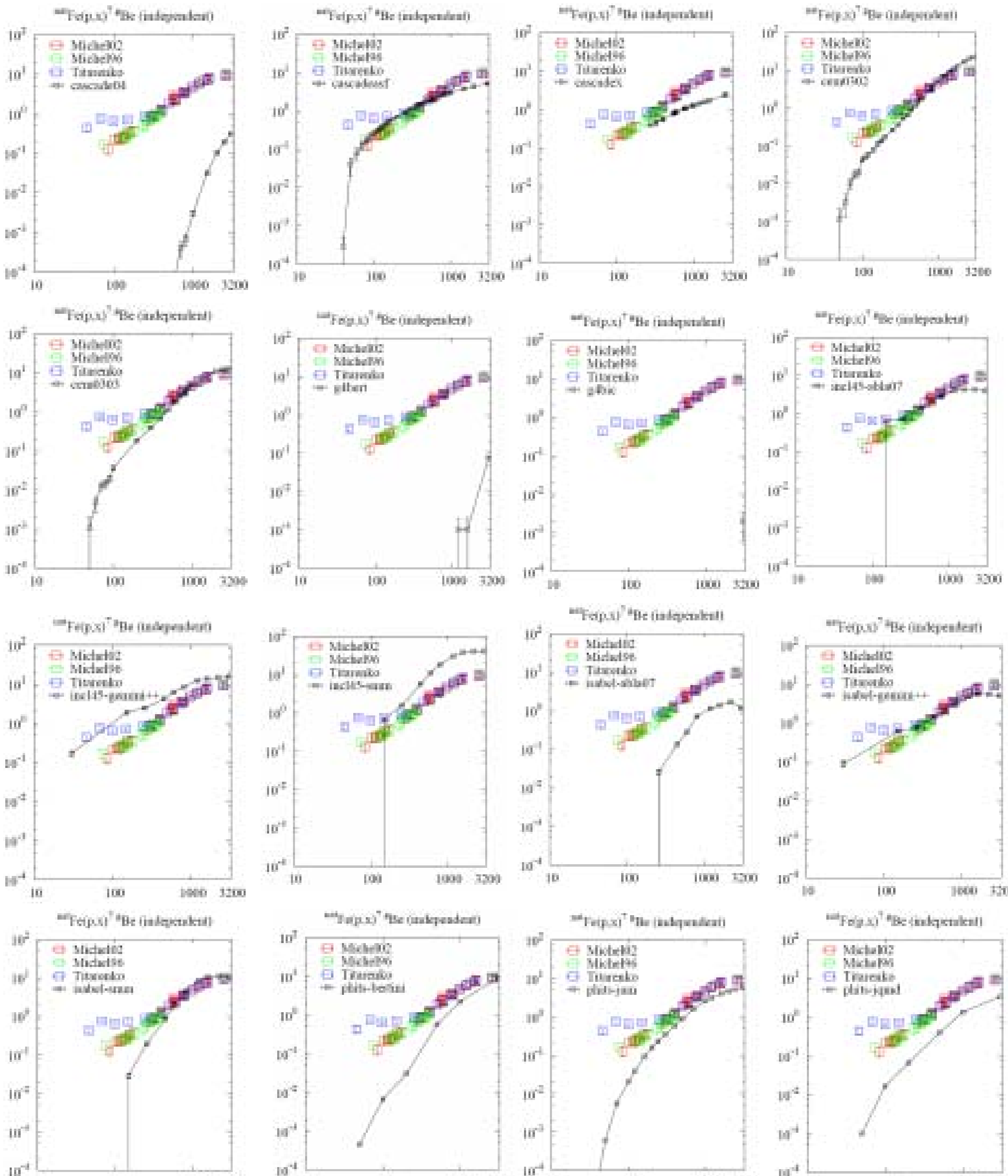
# He-4 from iron



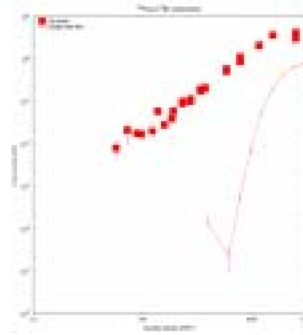


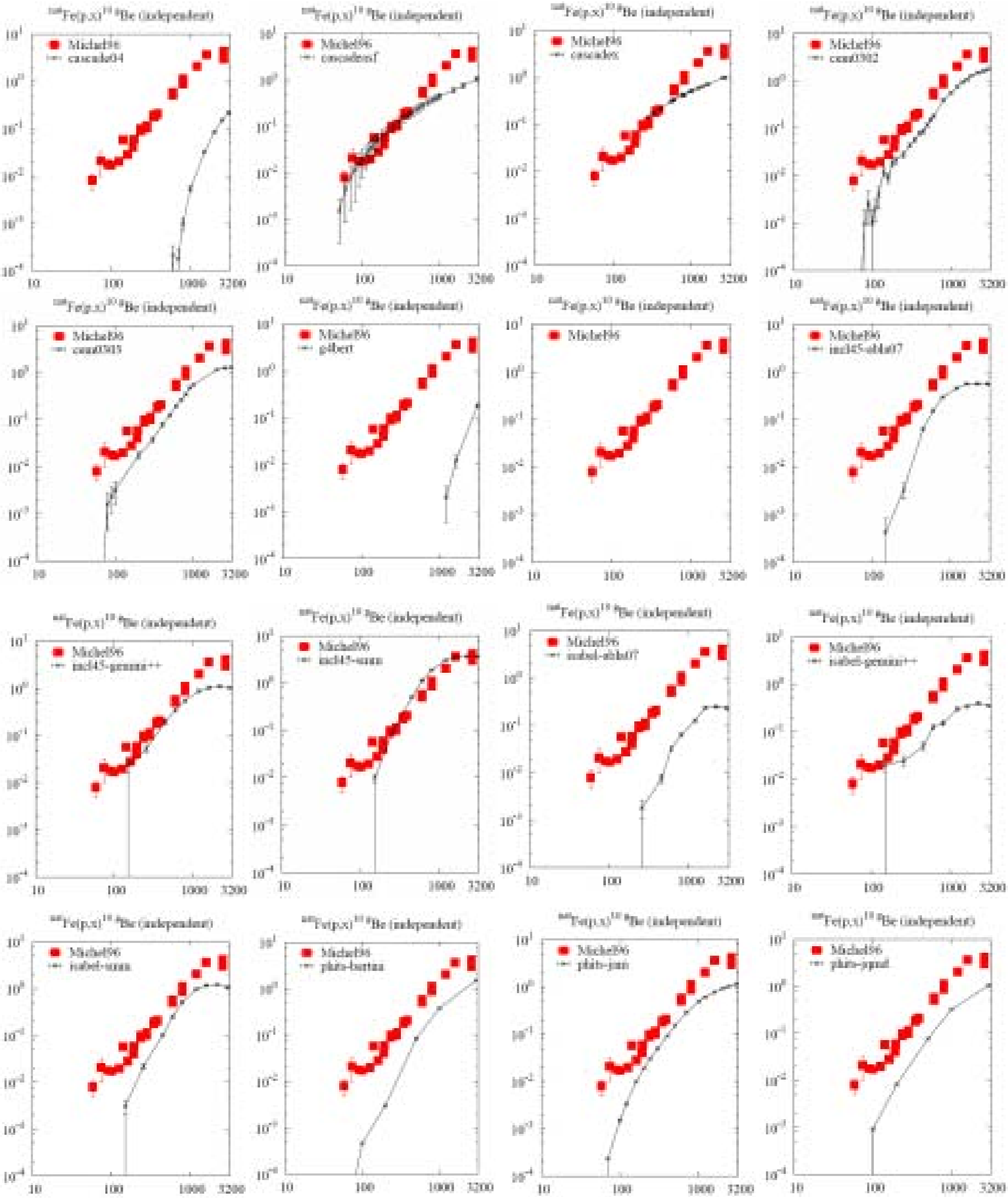
# Be-7 from iron



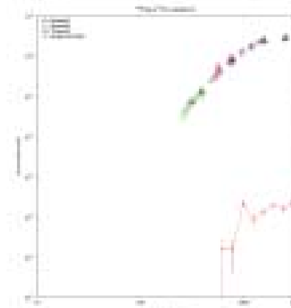


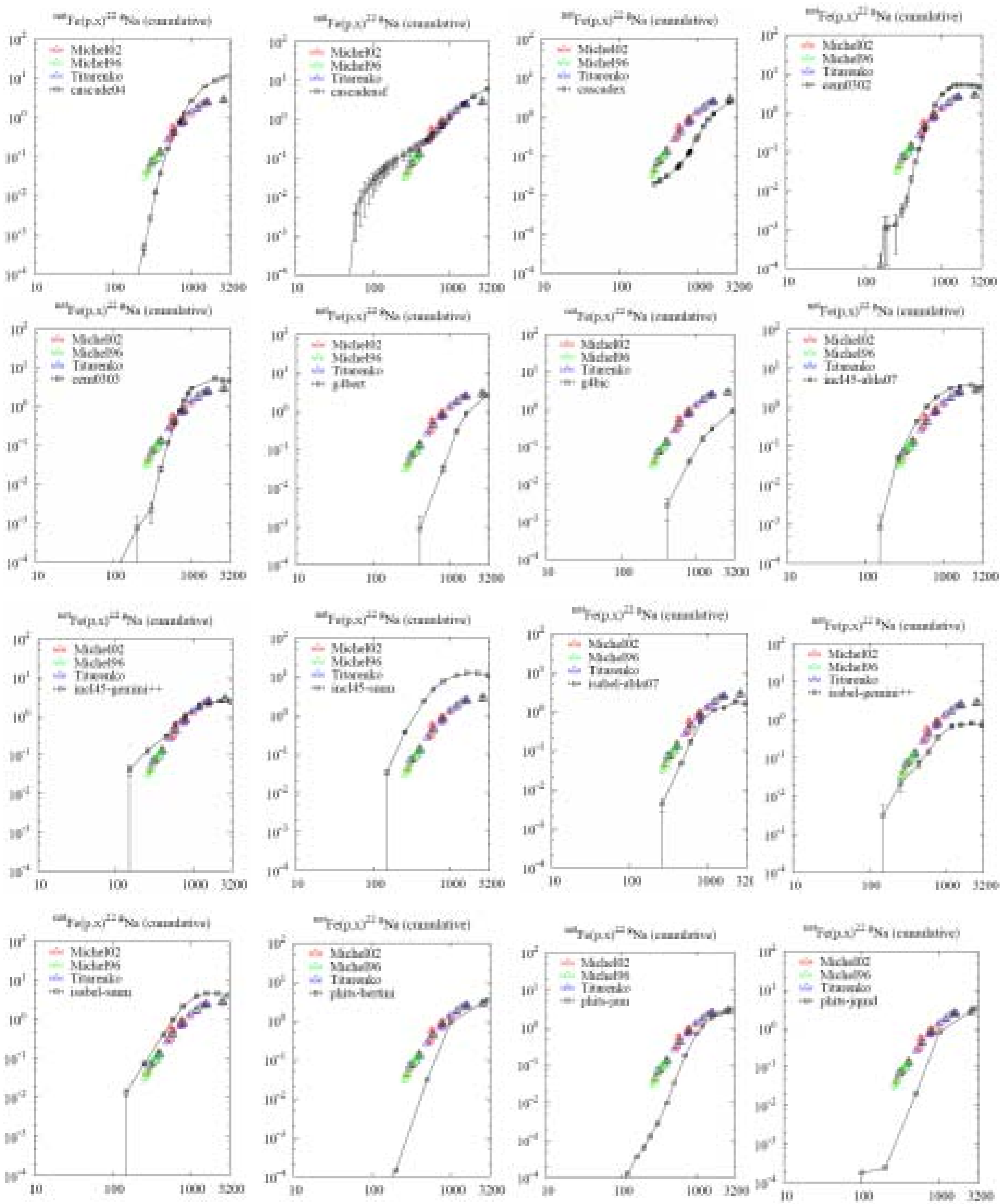
# Be-10 from iron



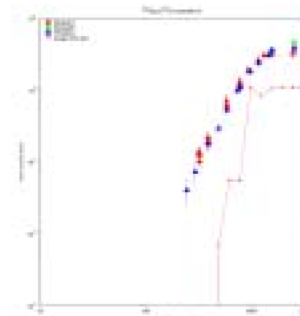


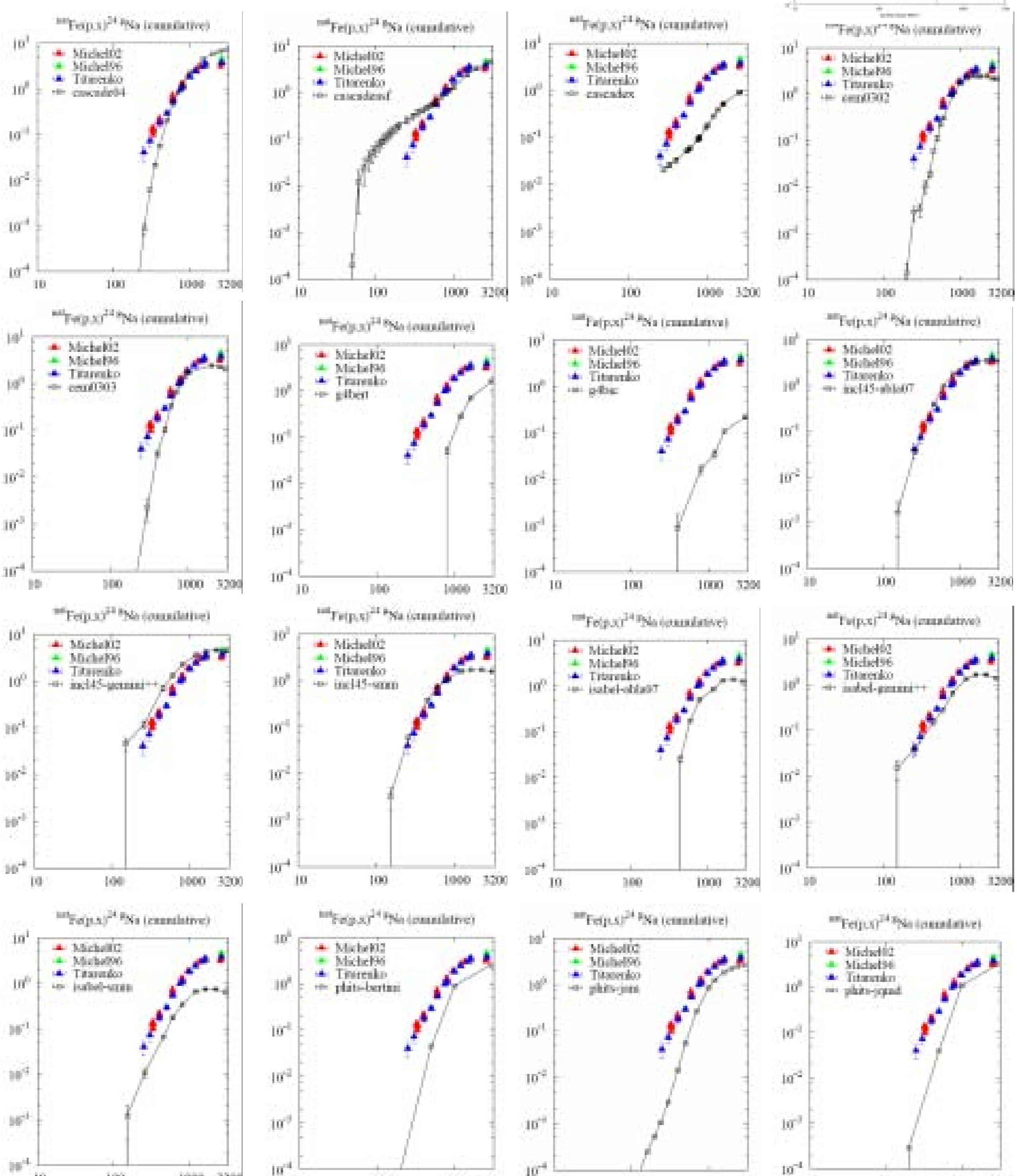
# Na-22 from iron



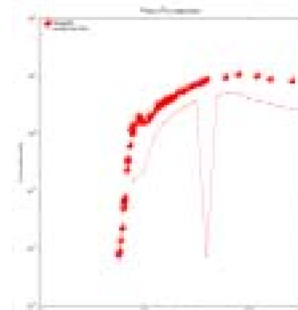


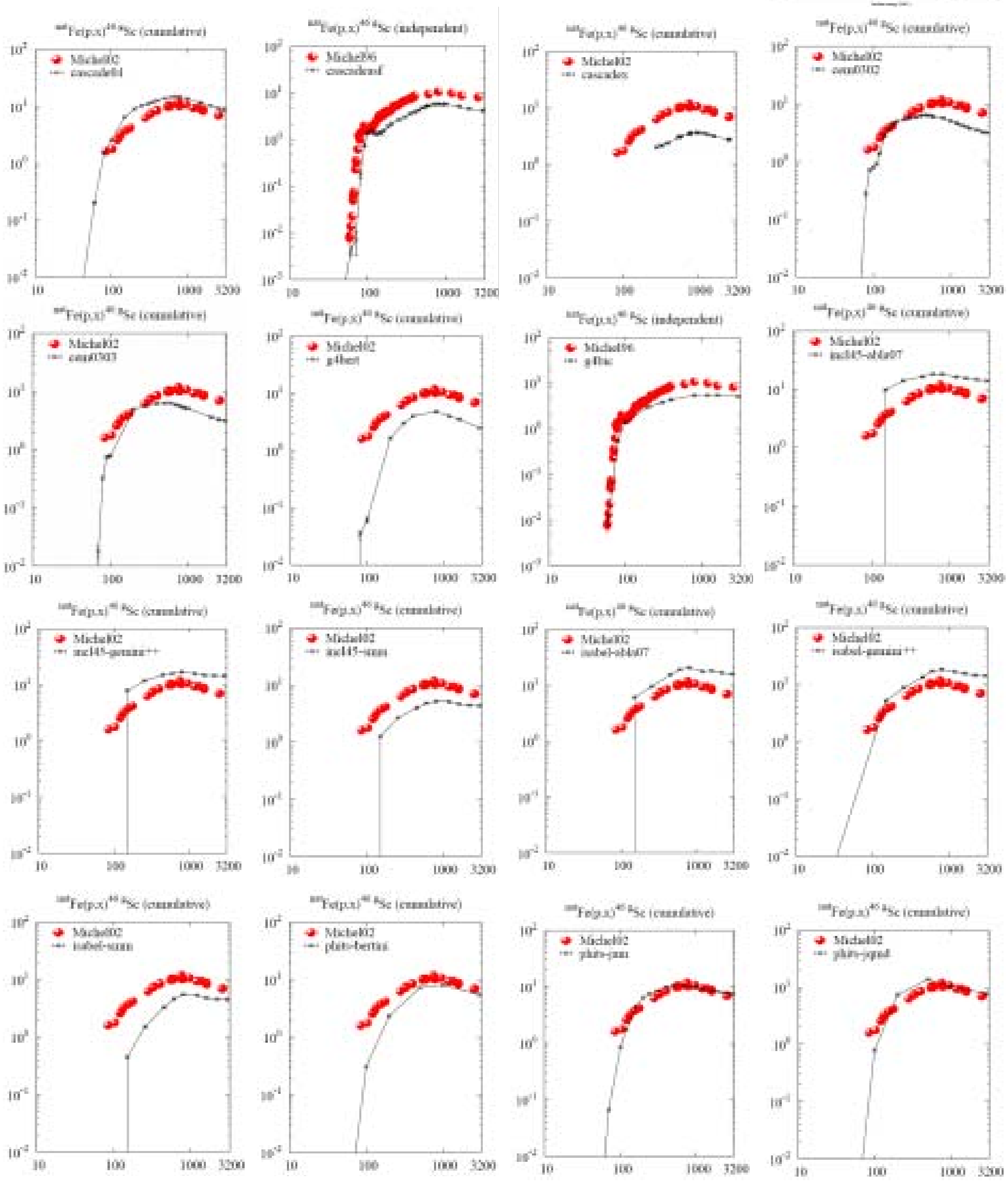
# Na-24 from iron



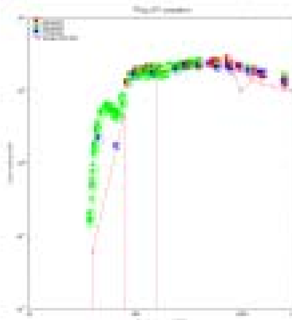


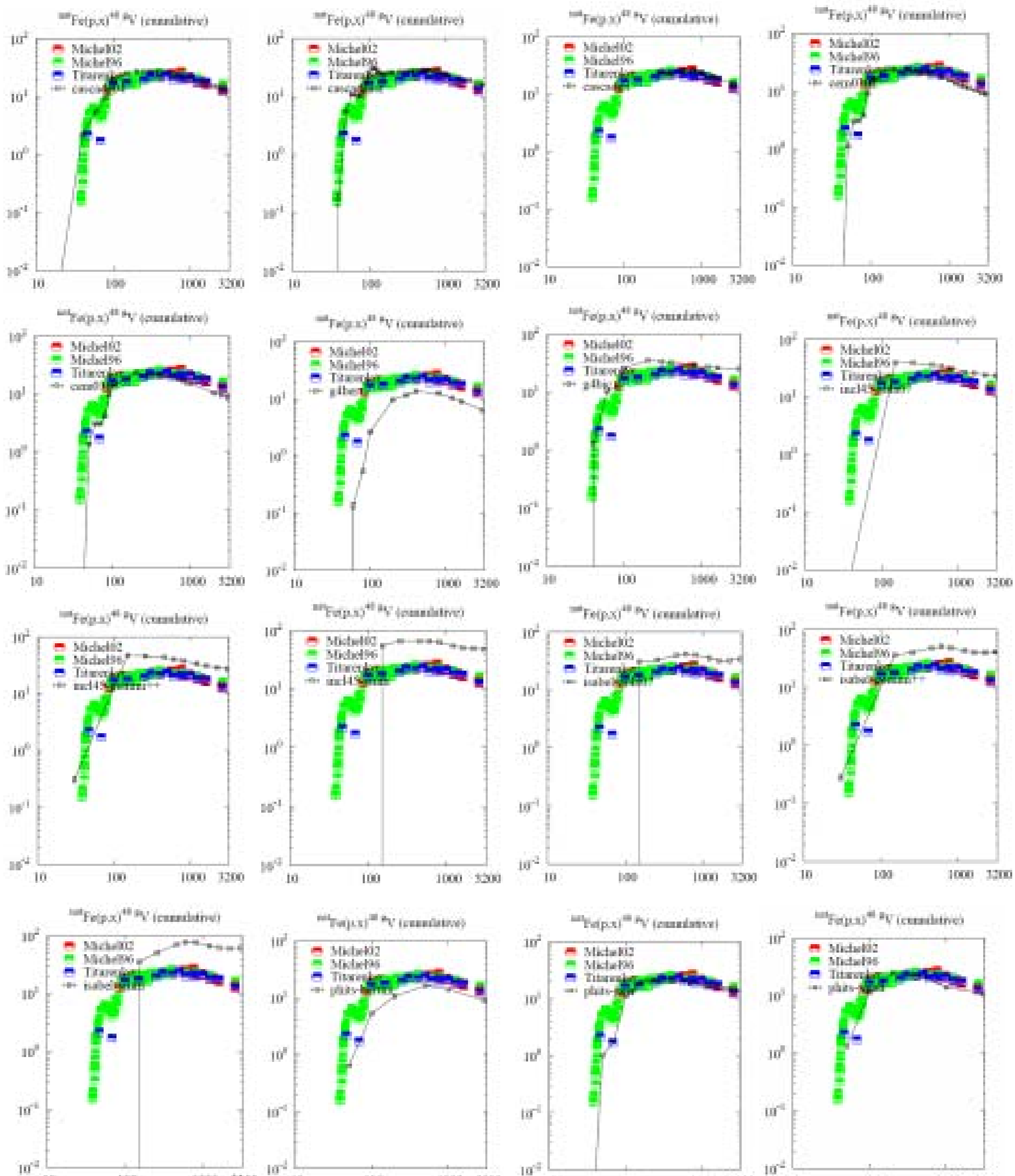
# Sc-46 from iron



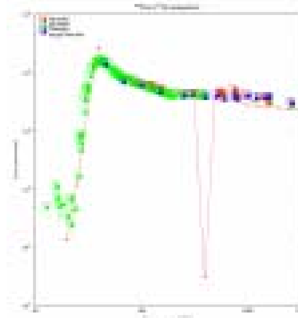


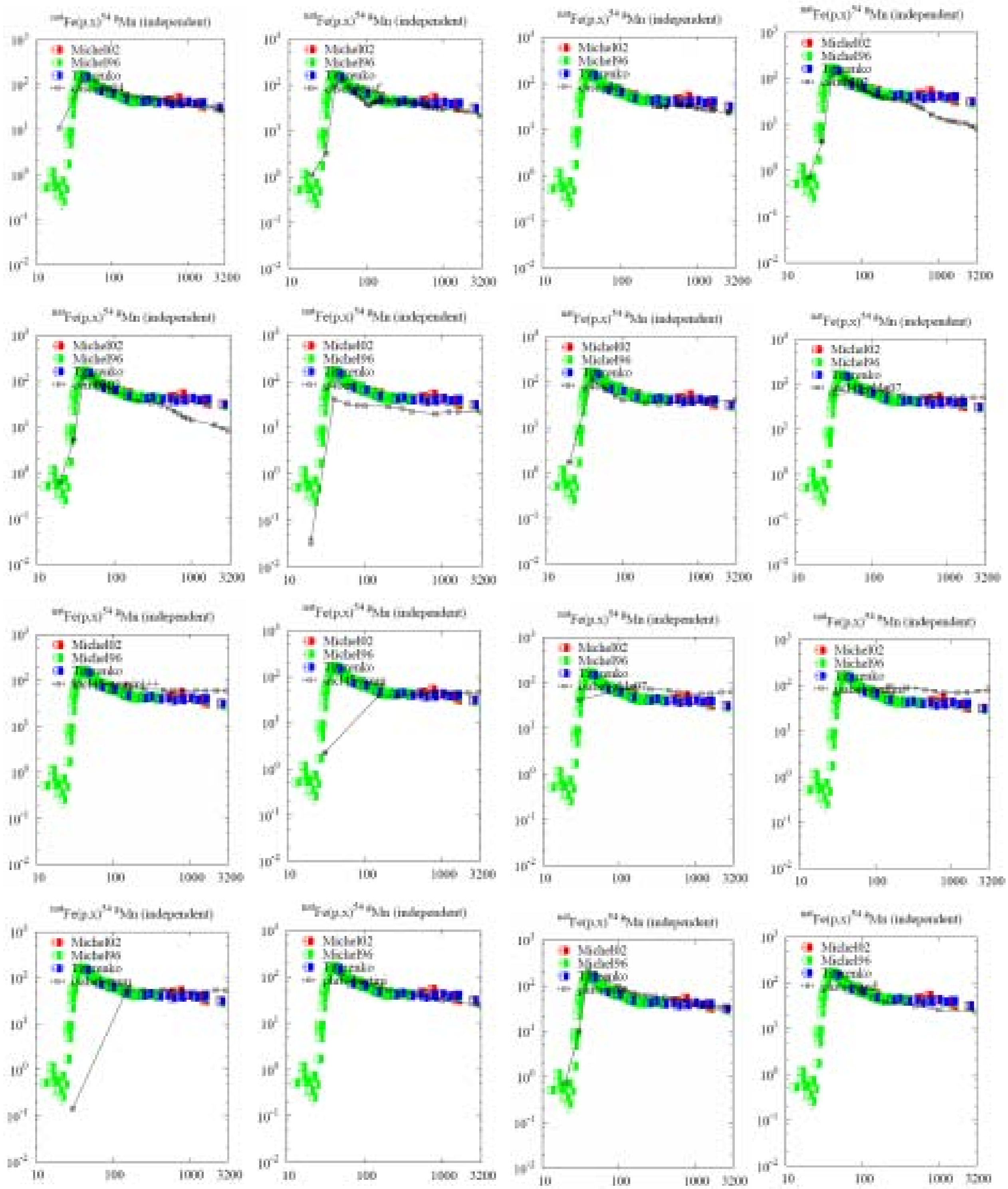
# V-48 from iron



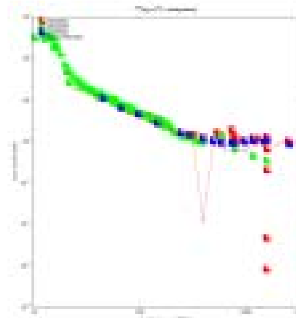


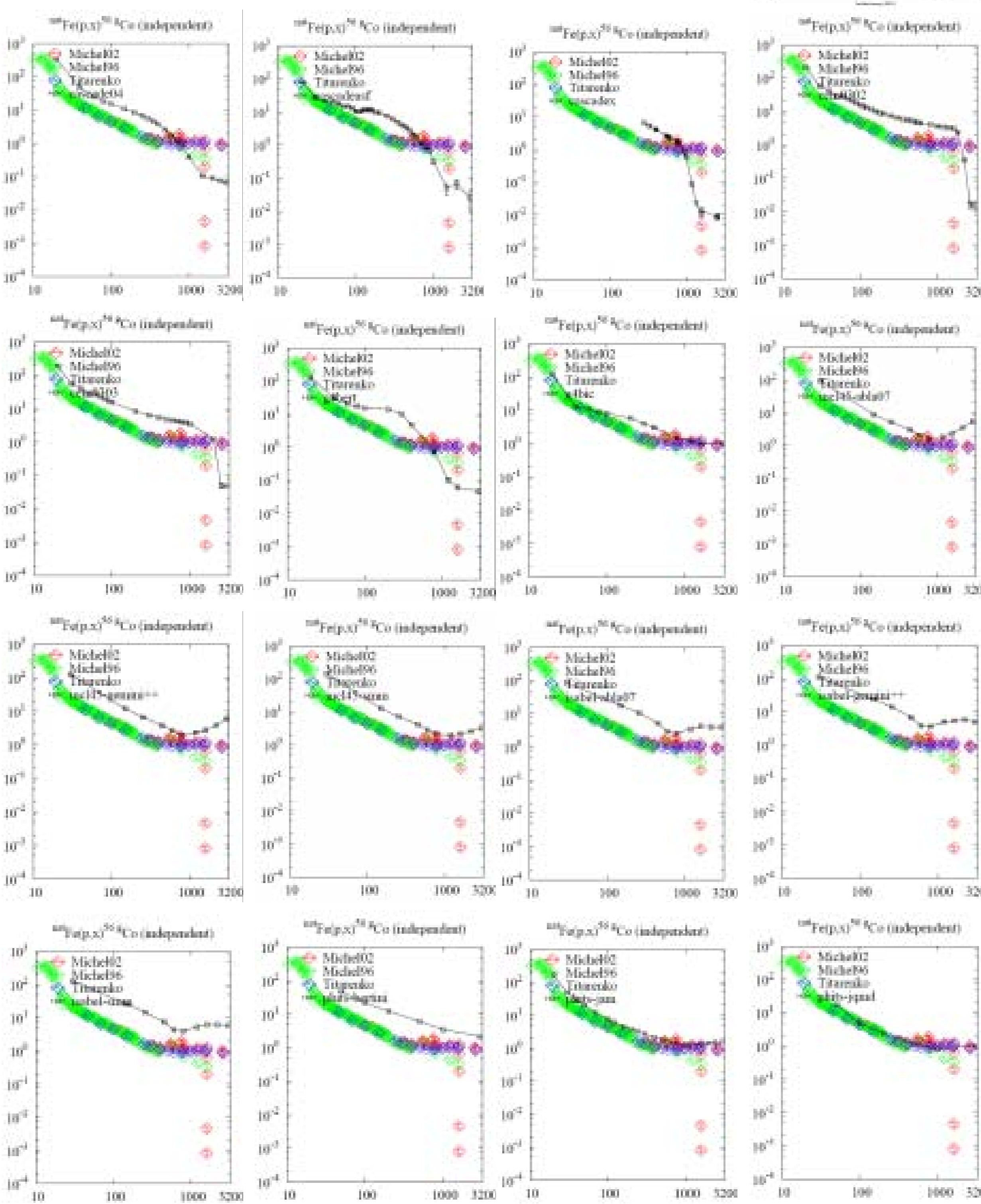
# Mn-54 from iron





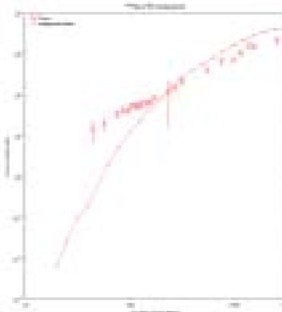
# Co-56 from Fe

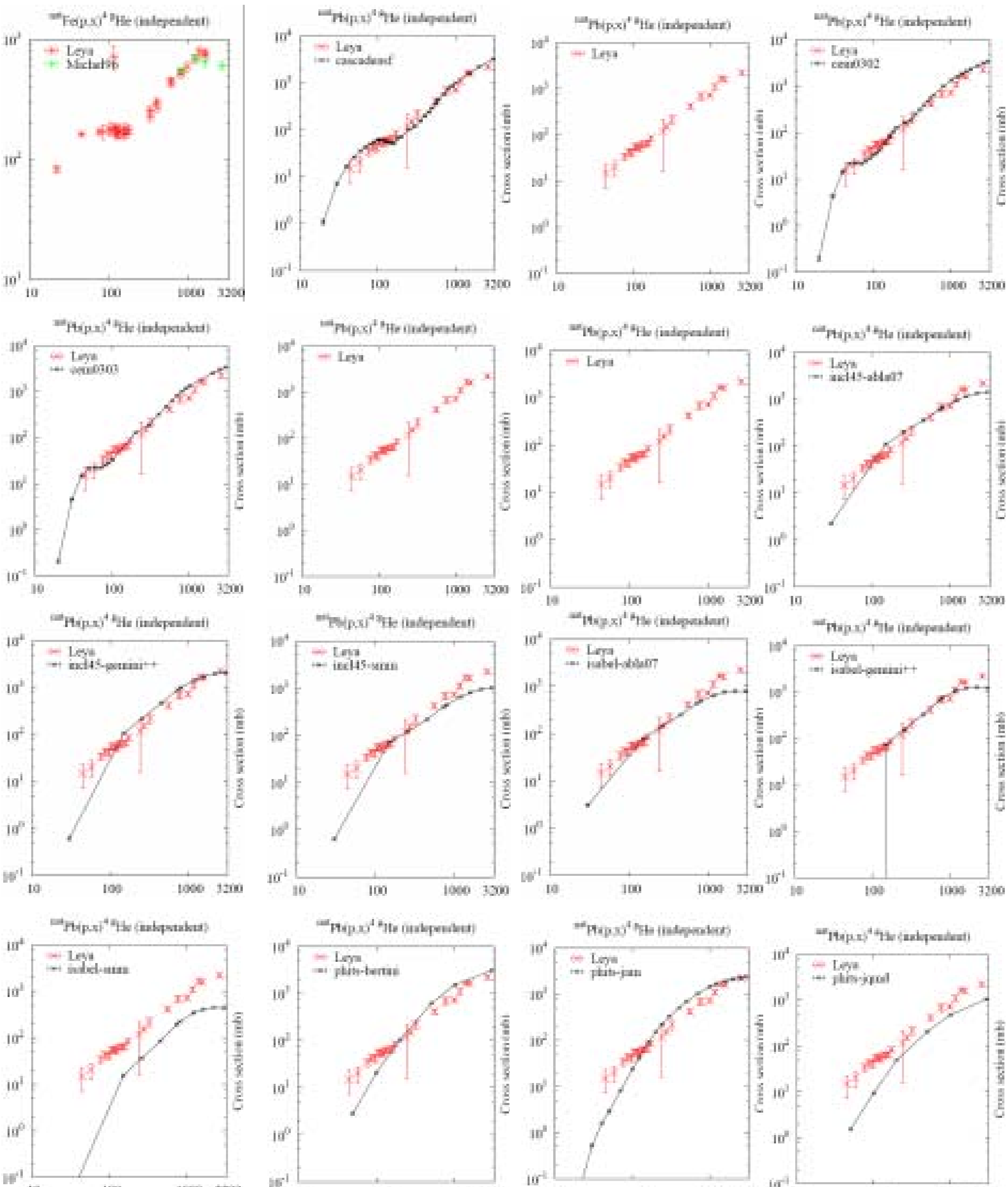



natFe(p,x)56Co (independent)
Michel02
Michel96
Titarenko
10
100
1000
3200

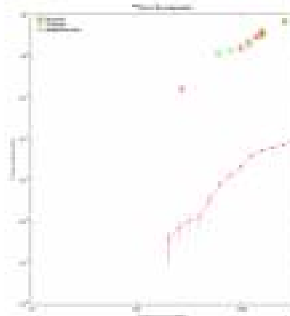
# Excitation Functions Lead

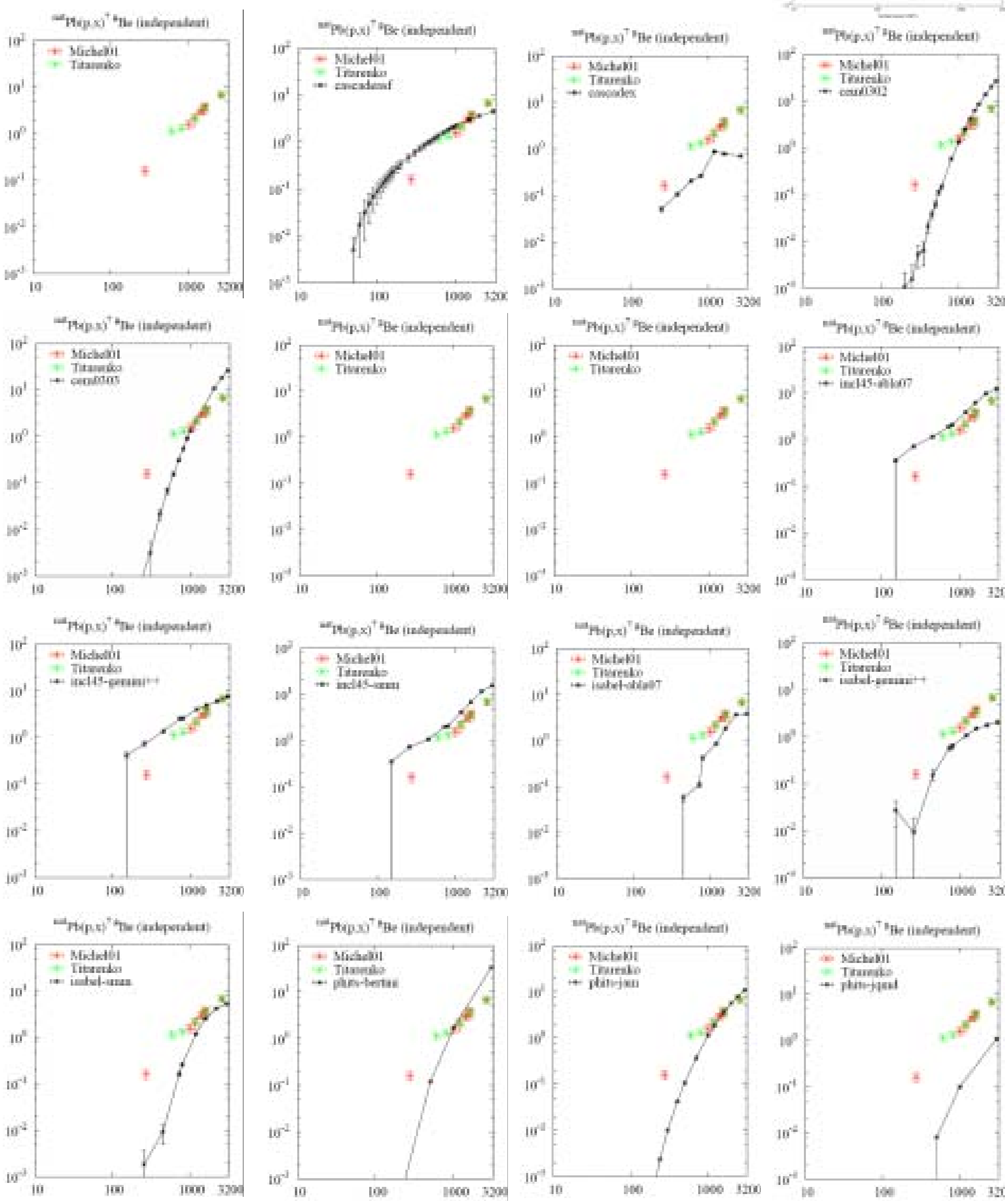
# He-4 from lead



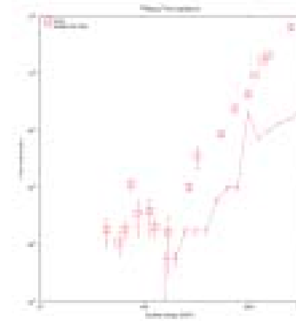


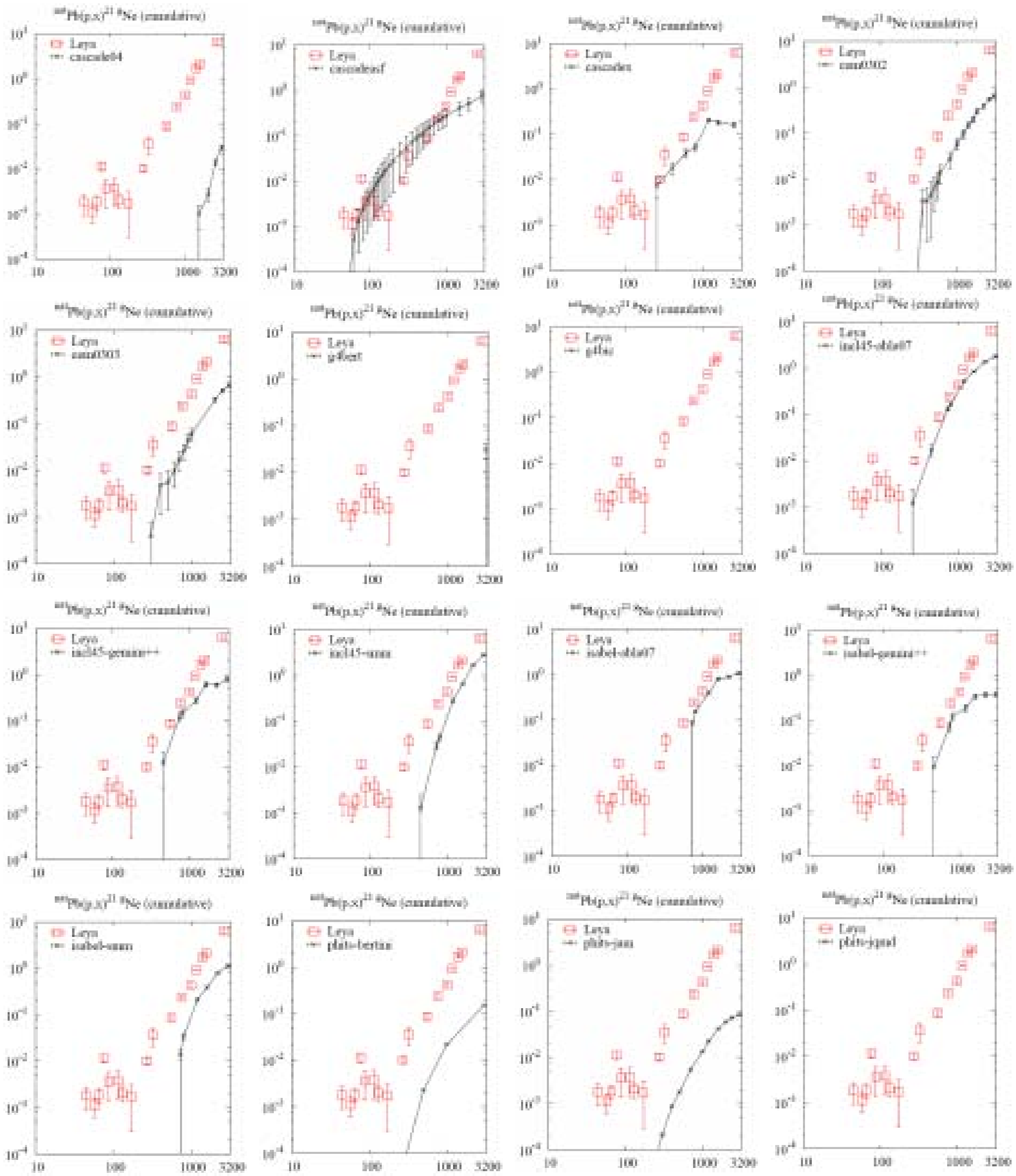
# Be-7 from lead



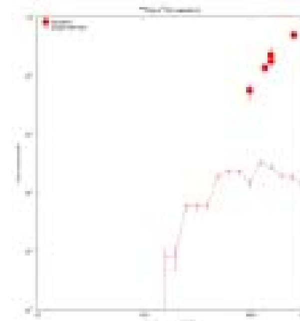


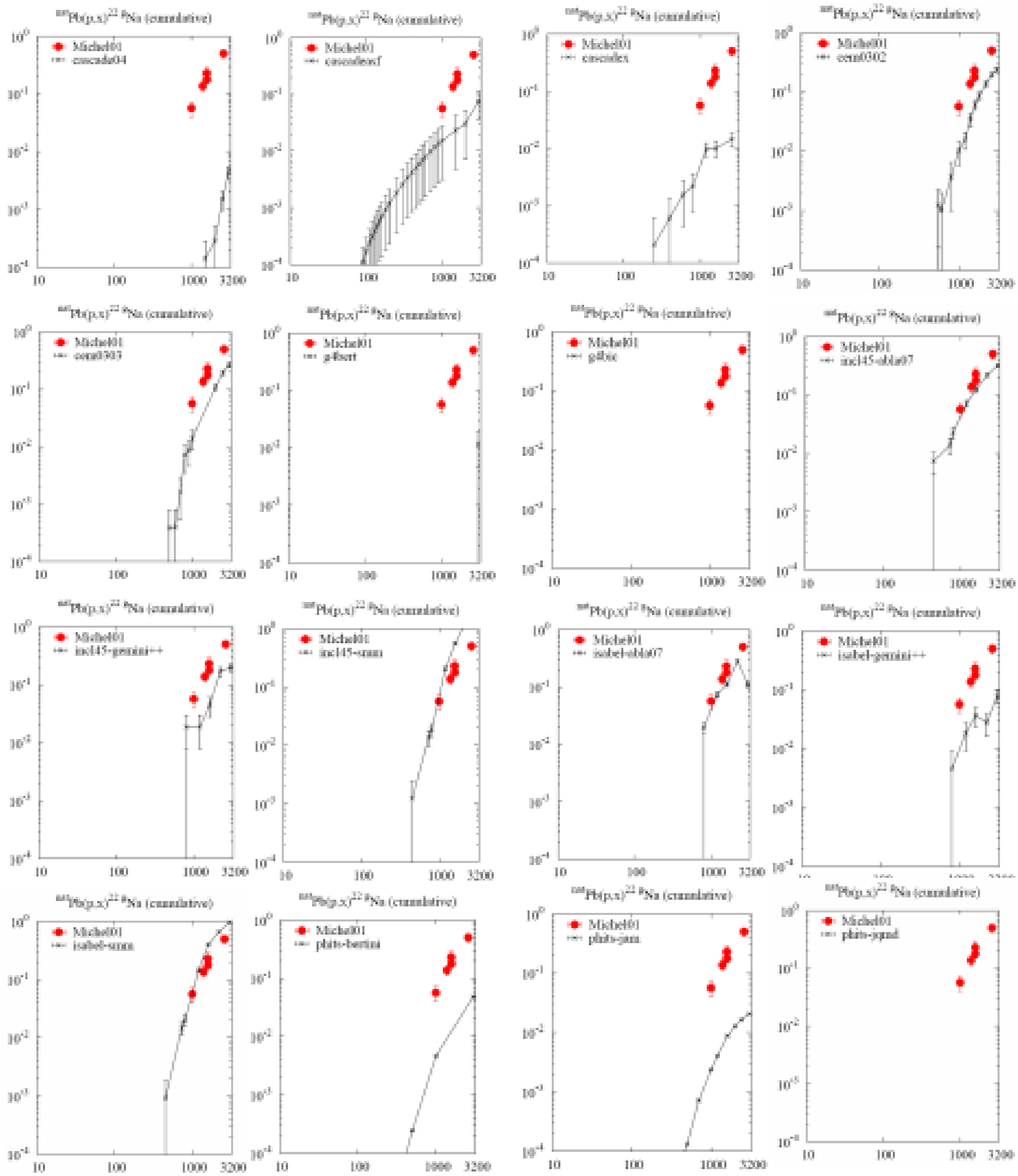
# Ne-21 from lead



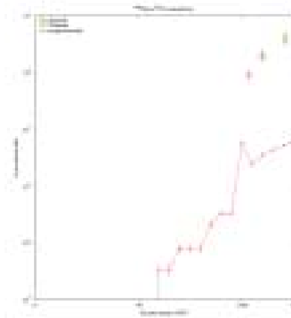


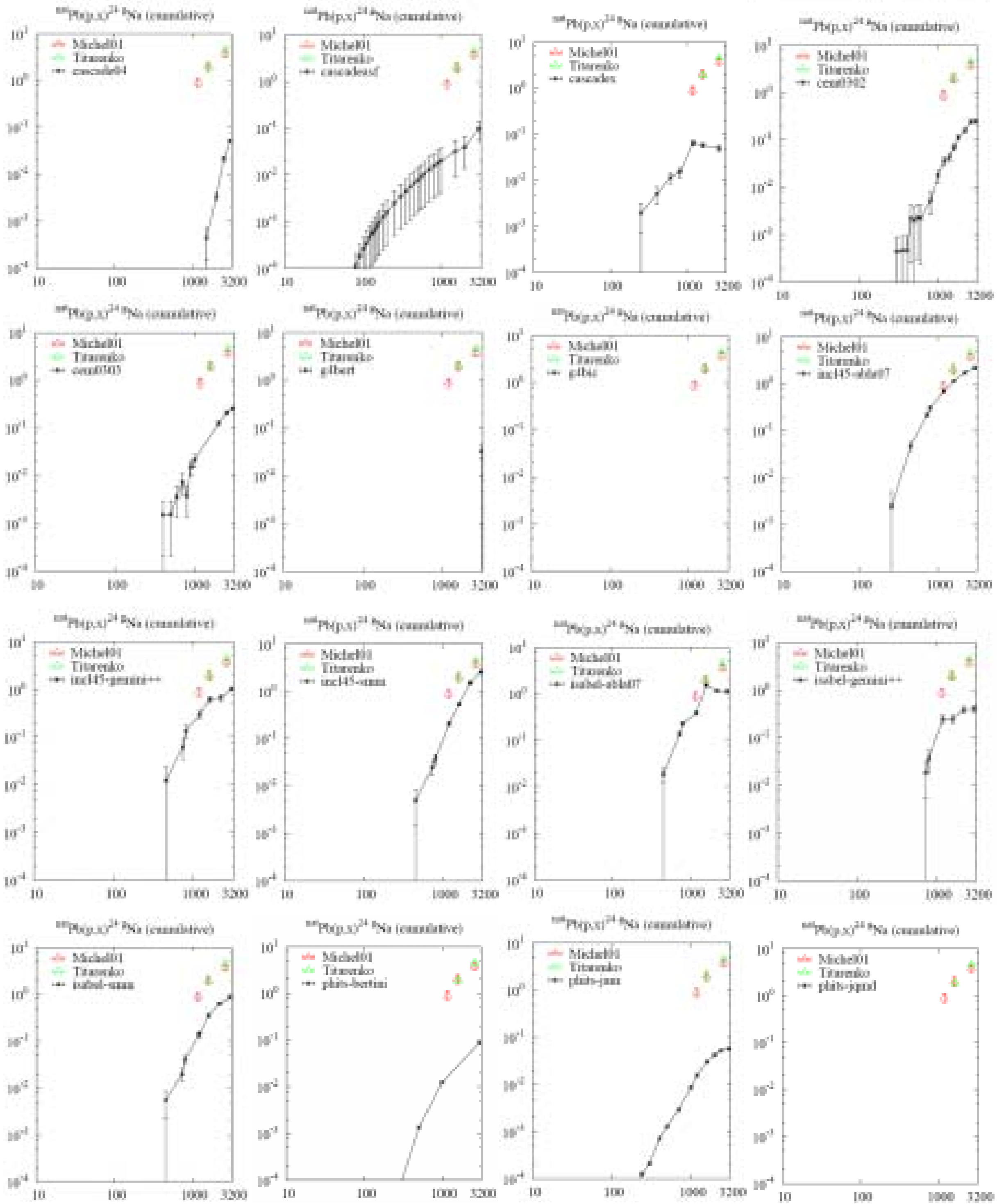
# Na-22 from lead



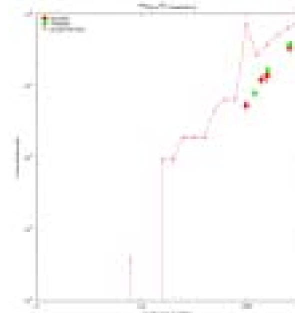


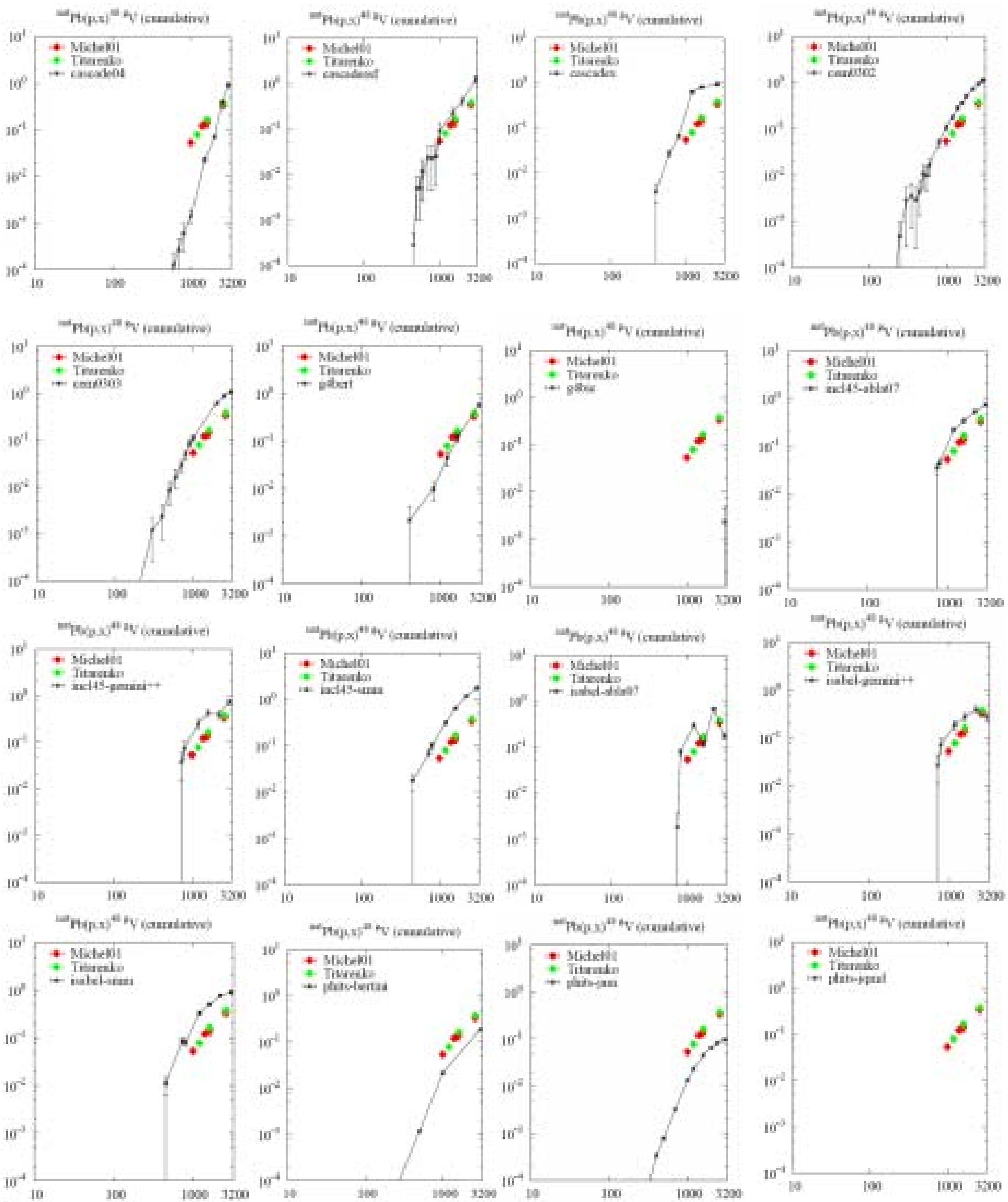
# Na-24 from lead



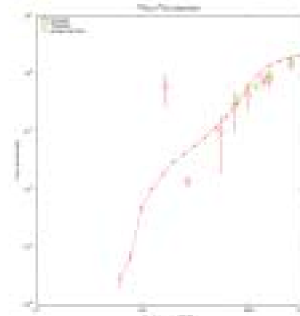


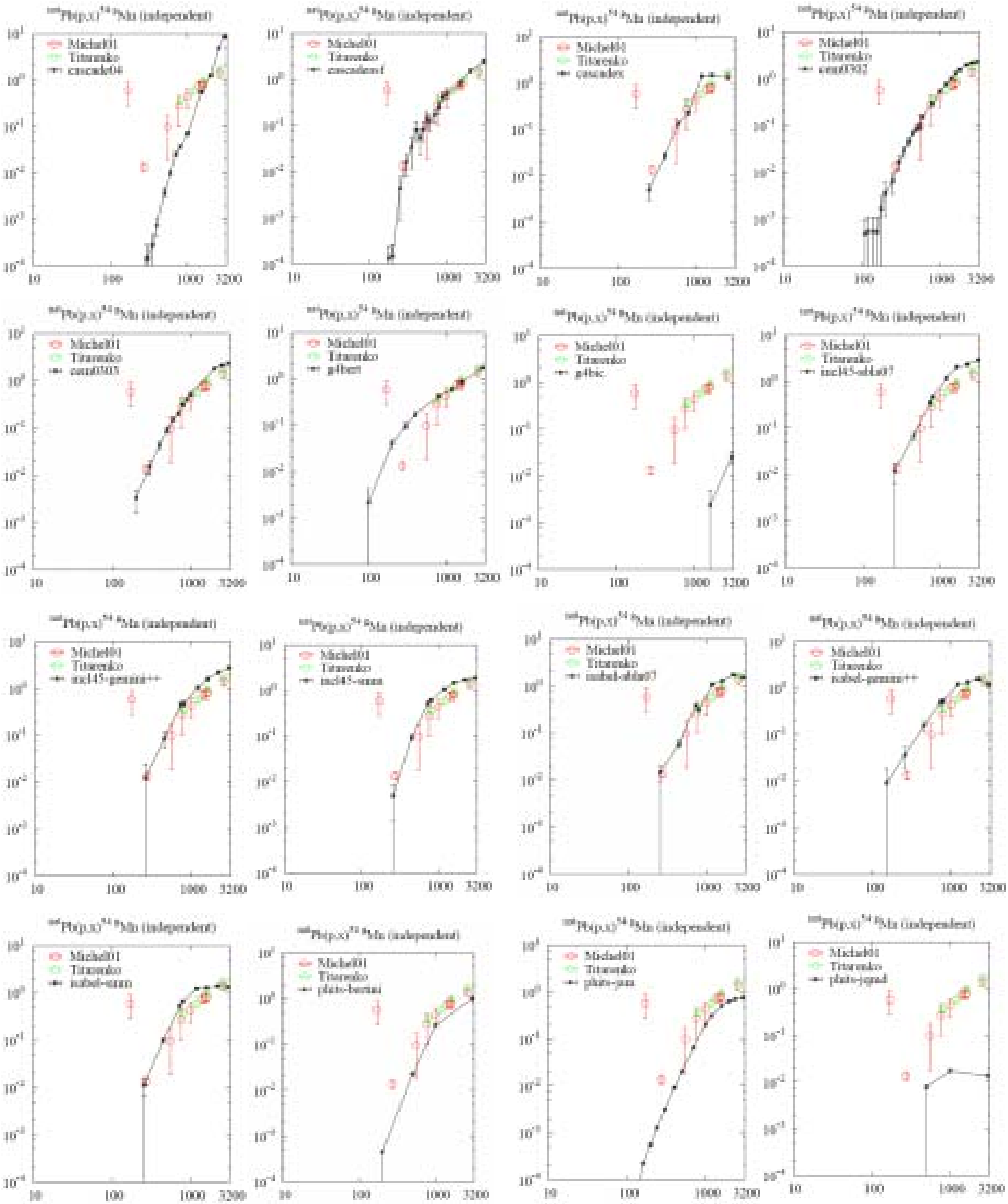
# V-48 from lead



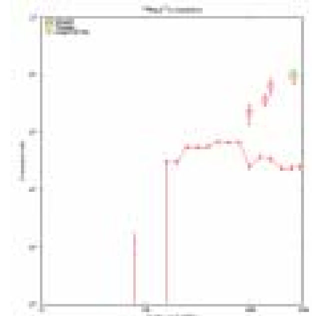


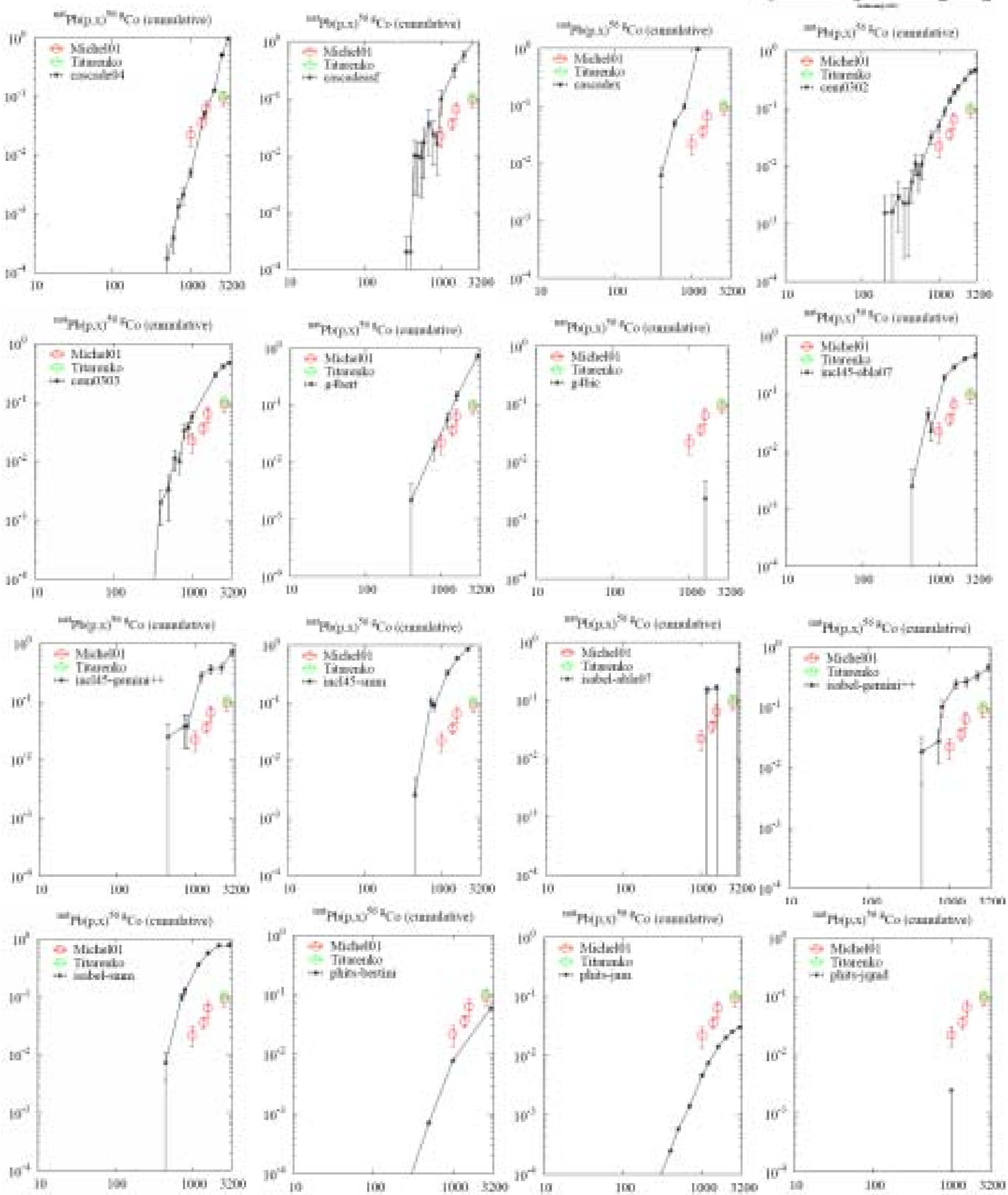
# Mn-54 from lead



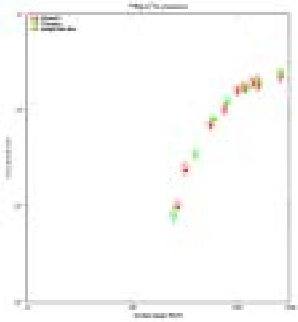


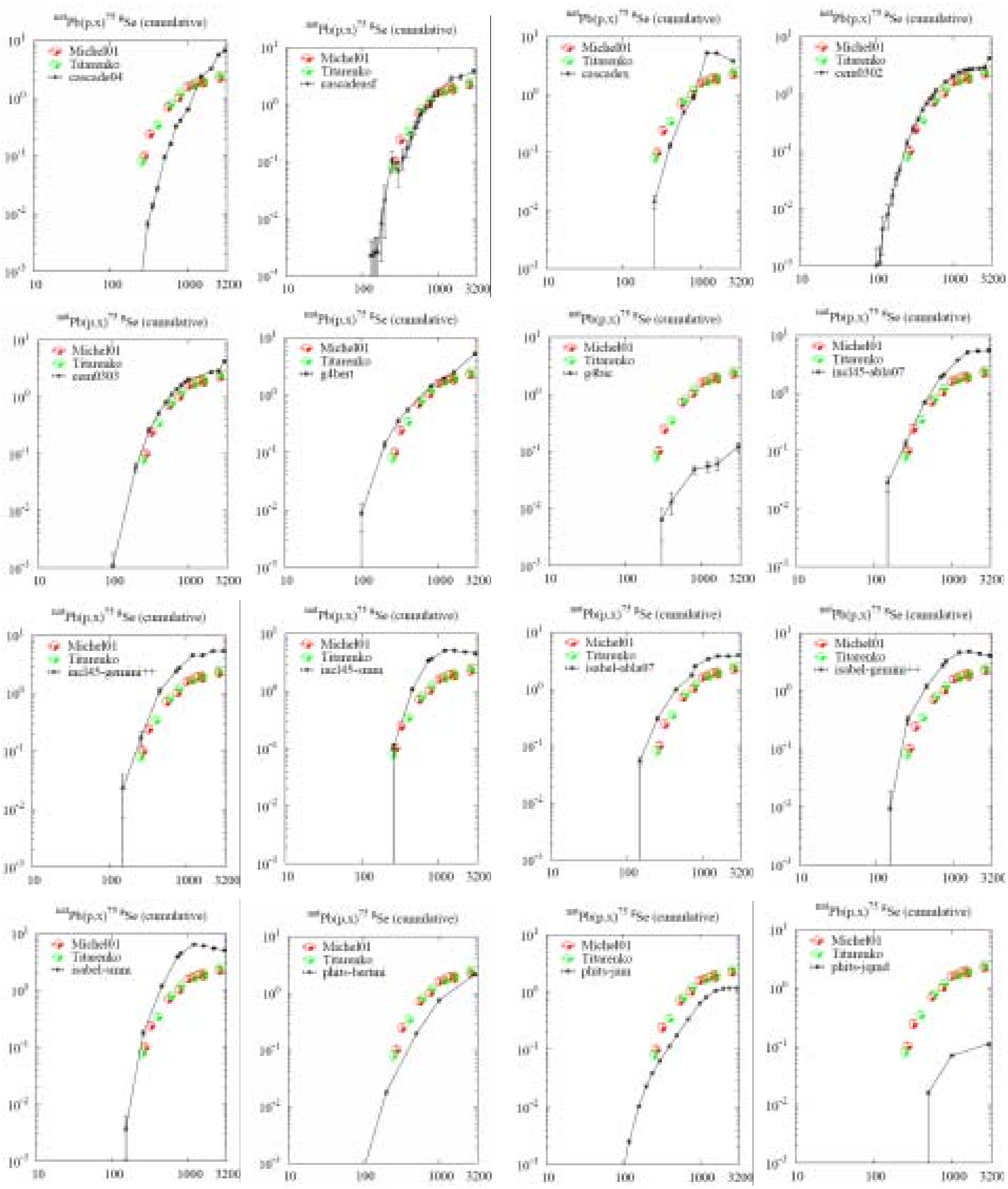
# Co-56 from lead



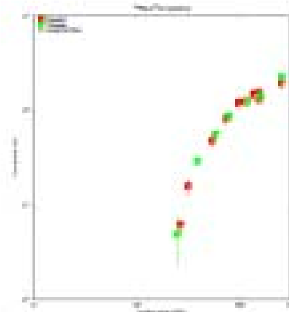


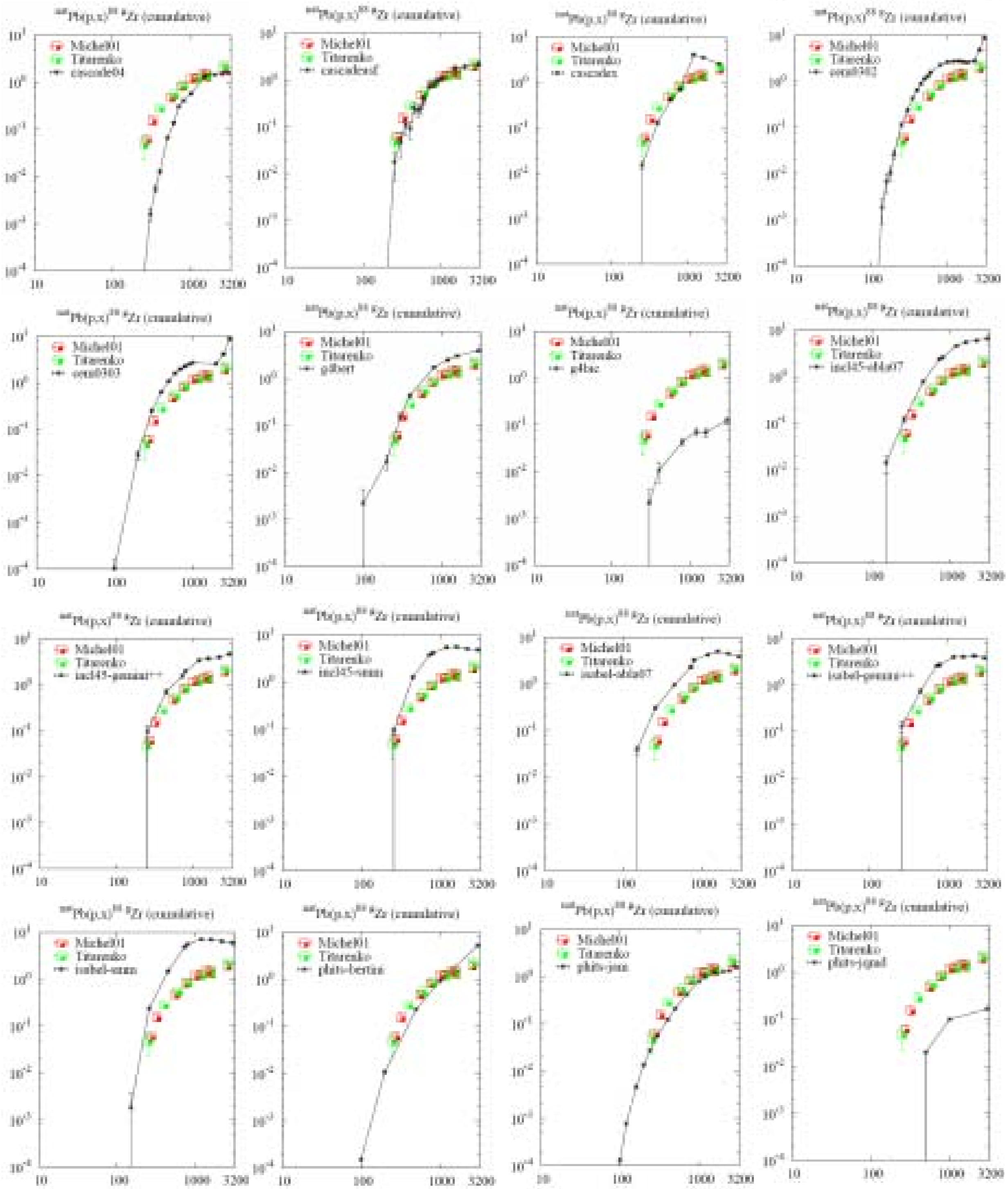
# Se-75 from lead



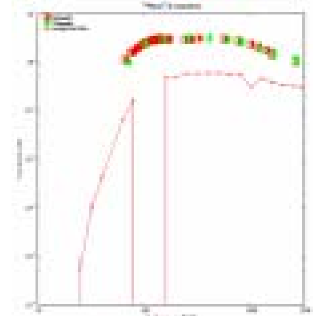


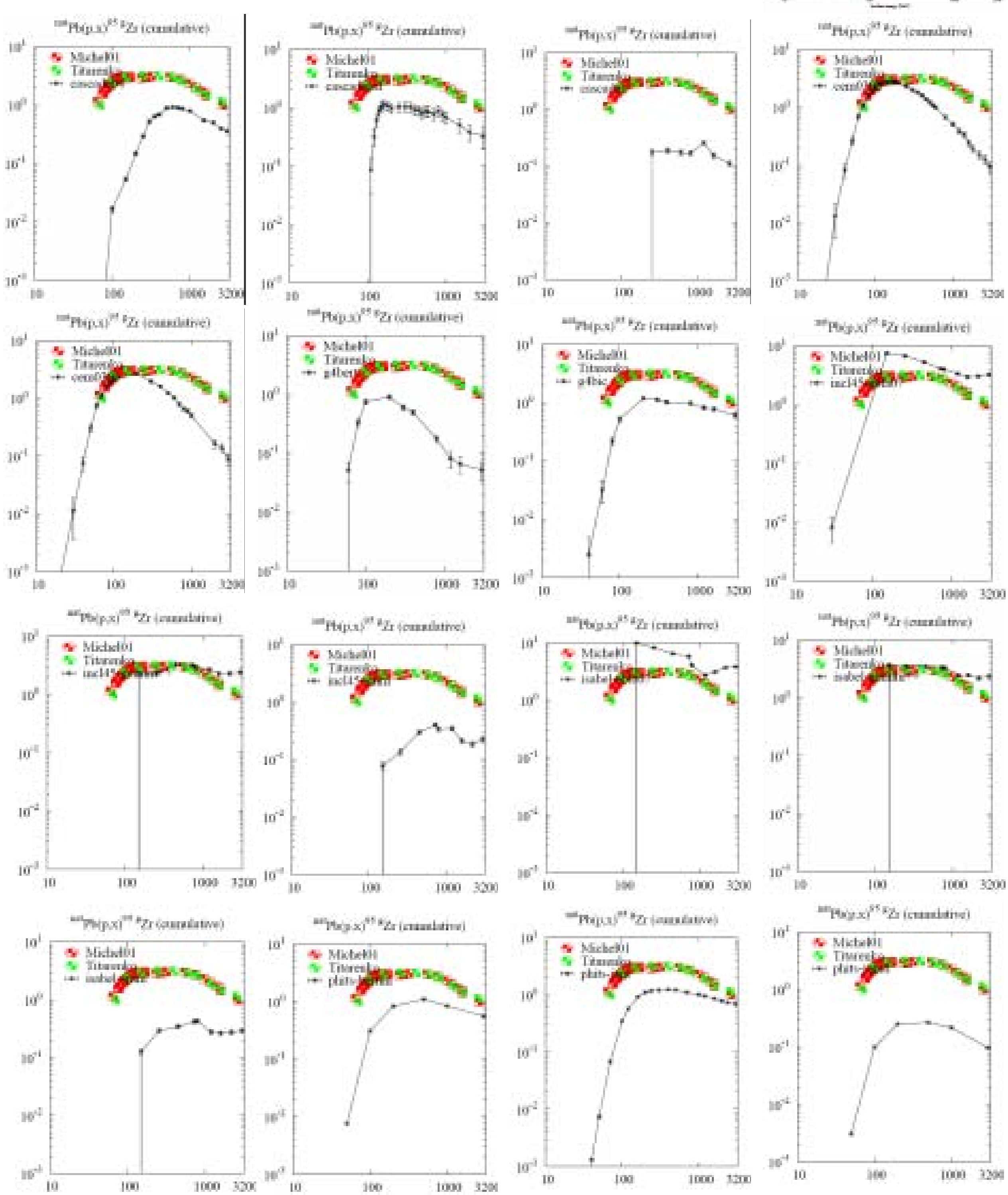
# Zr-88 from lead



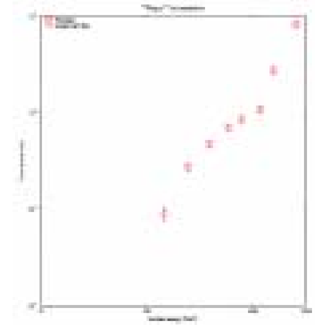


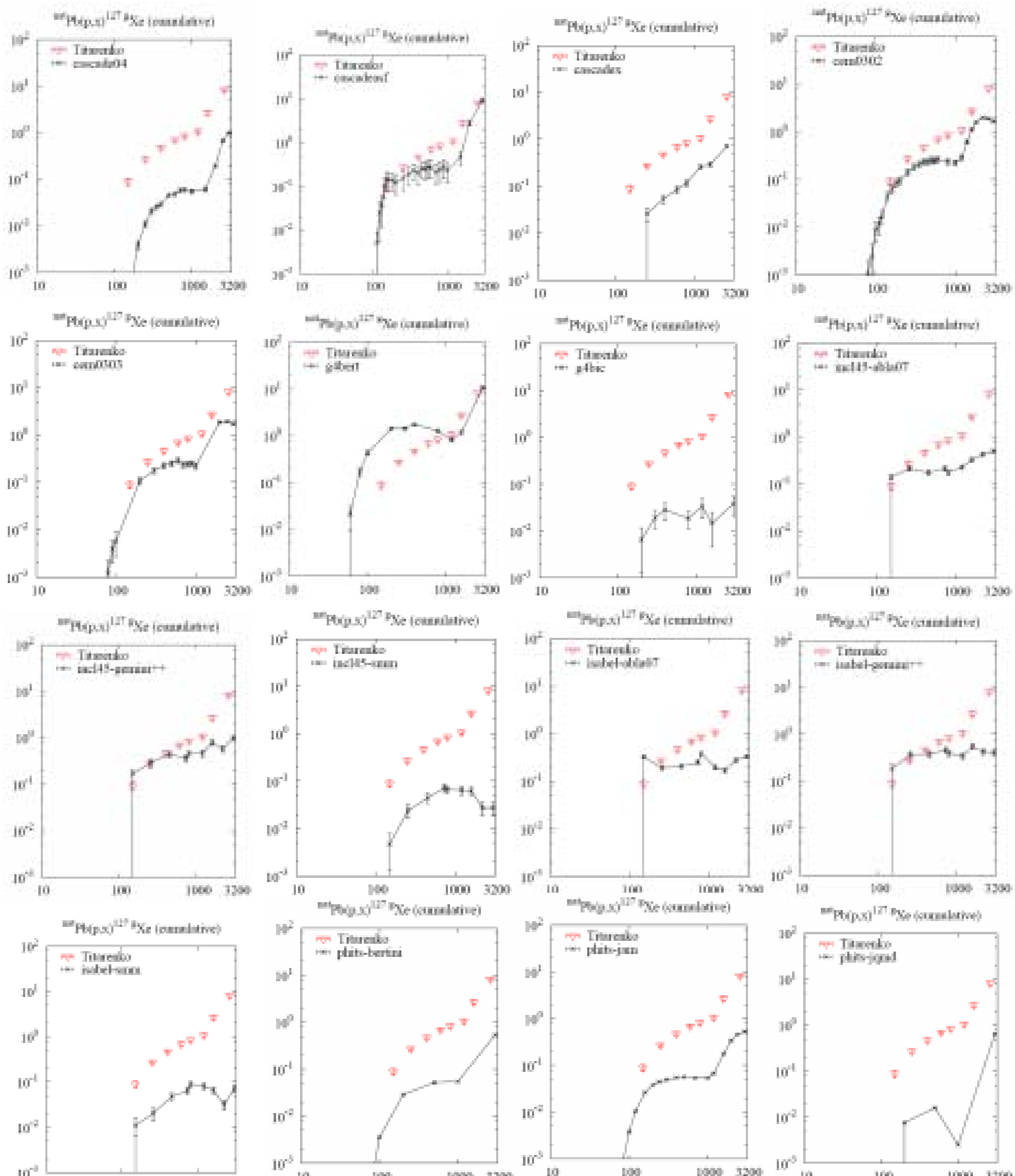
# Zr-95 from lead



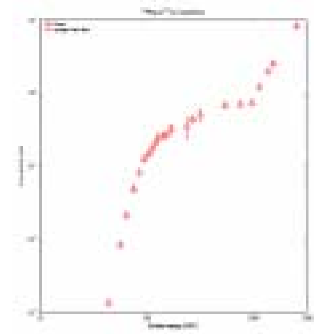


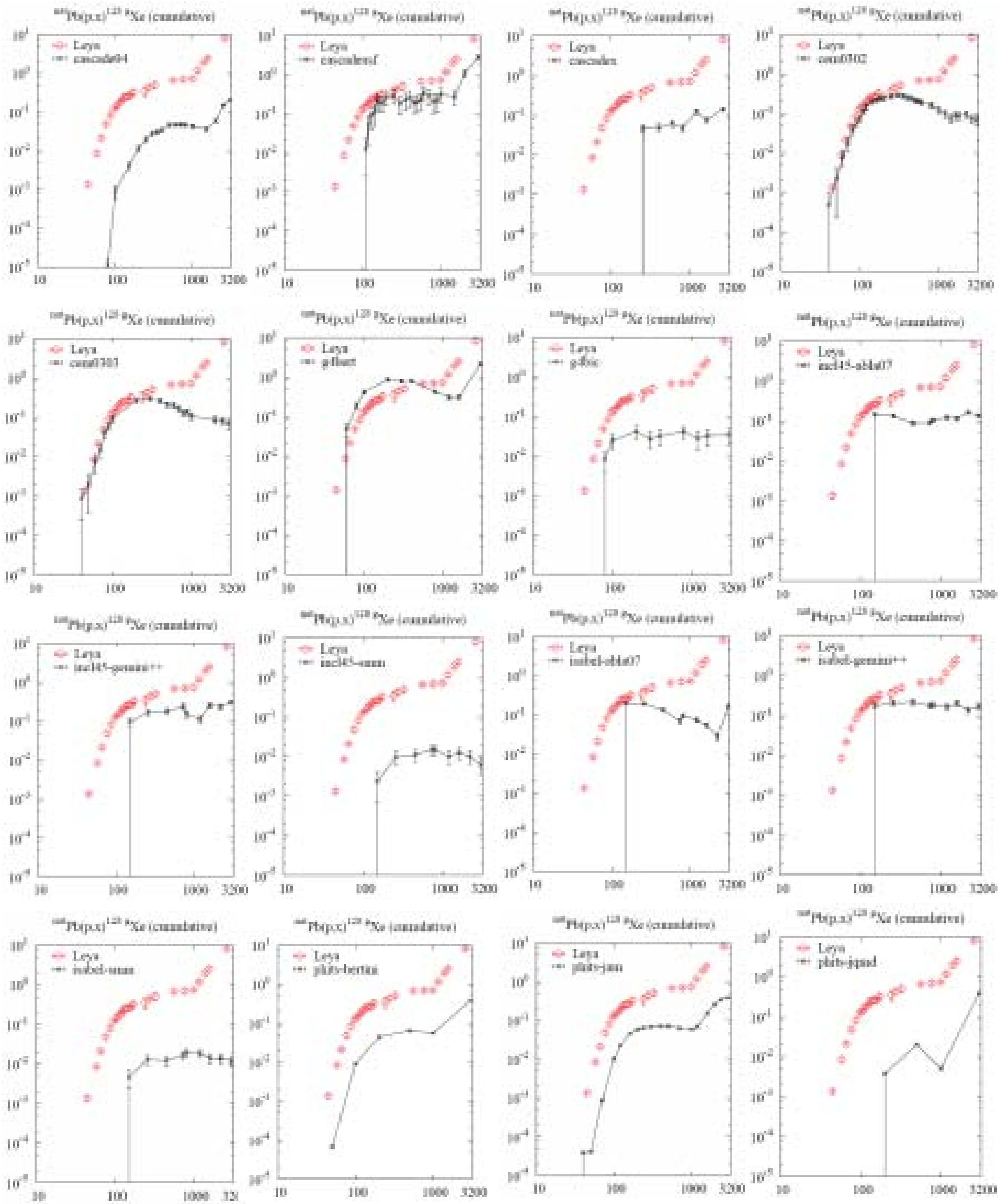
# Xe-127 from lead



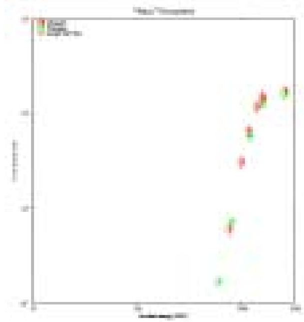


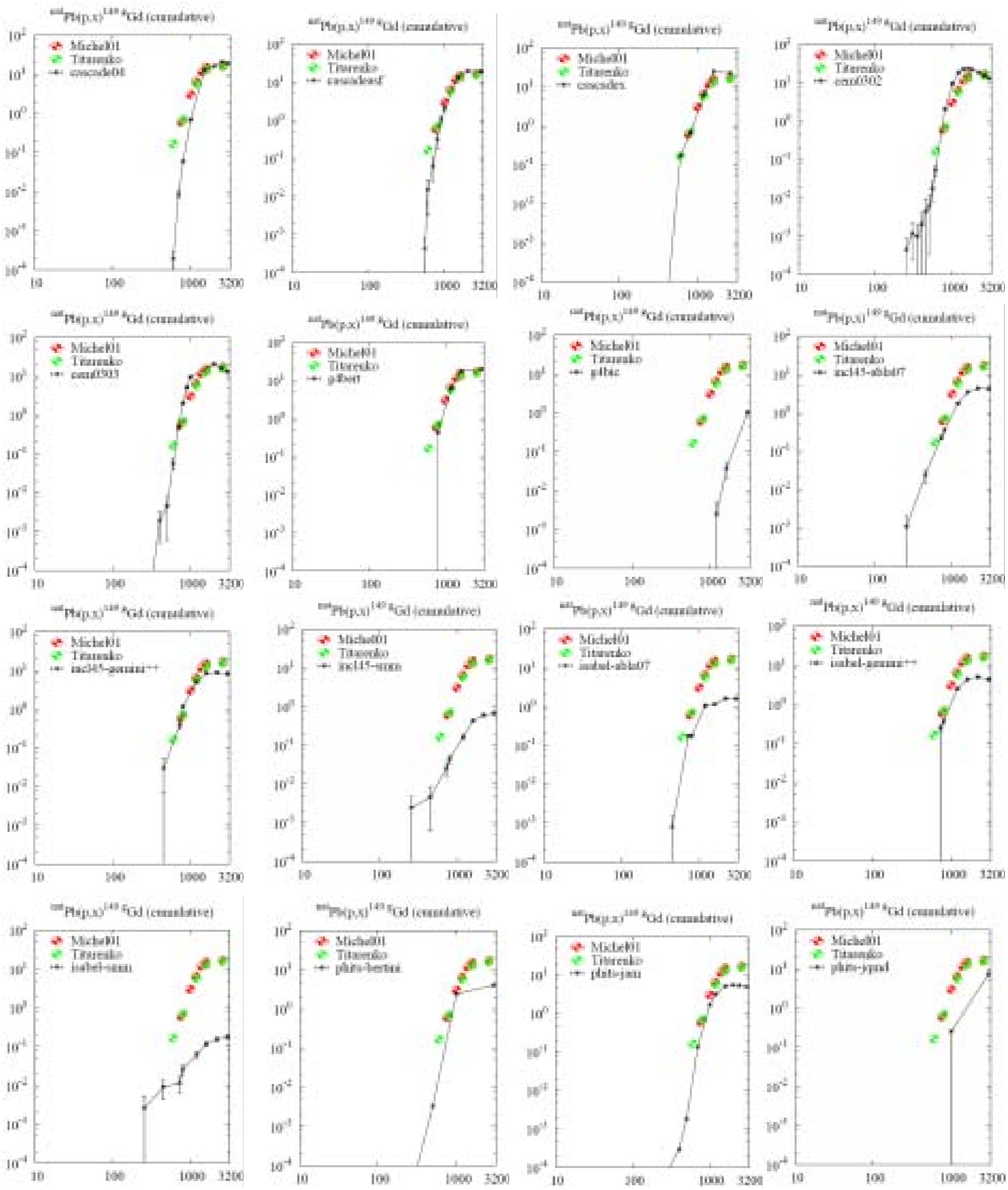
# Xe-128 from lead



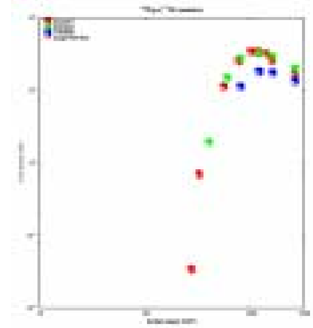


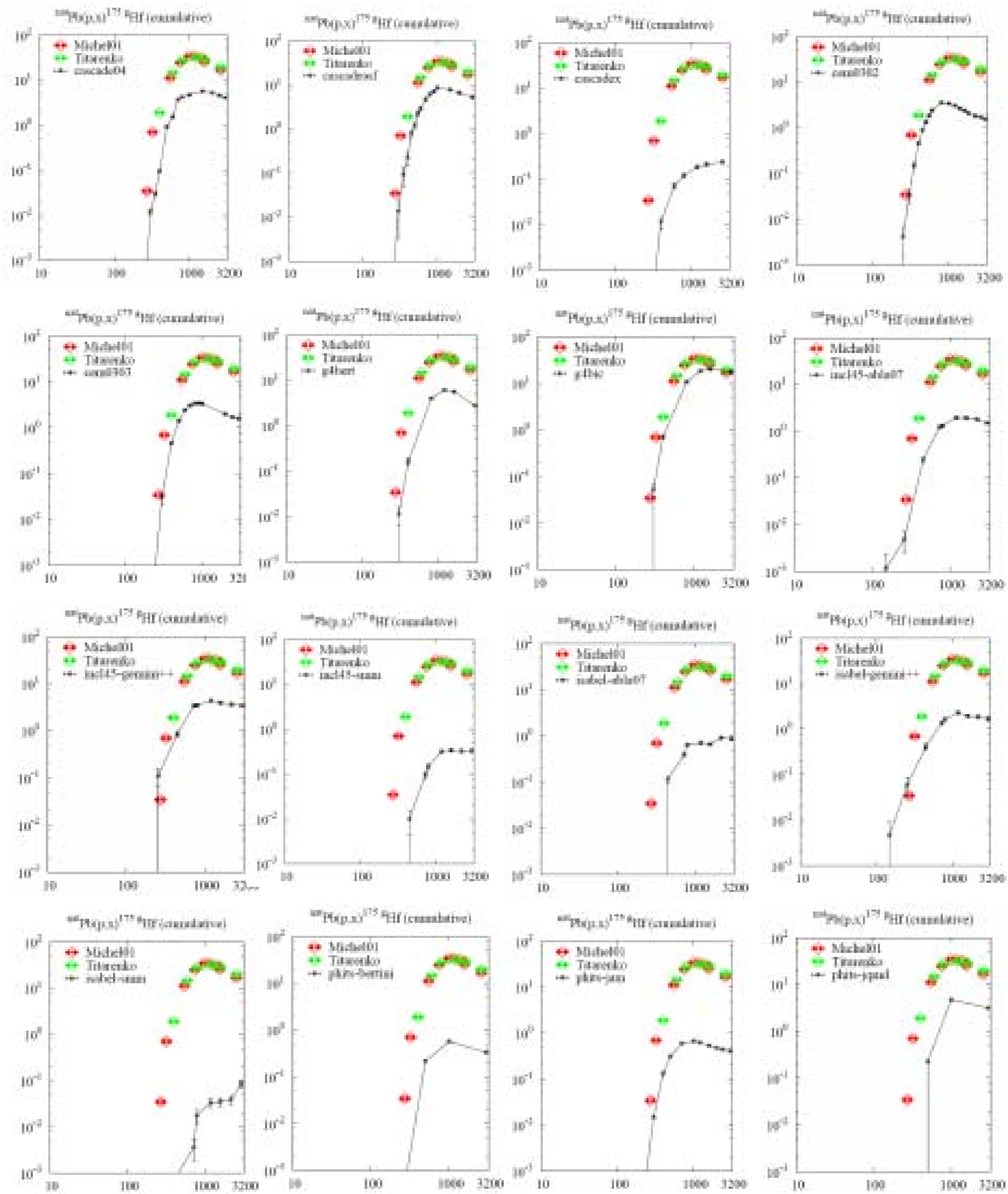
# Gd-149 from lead



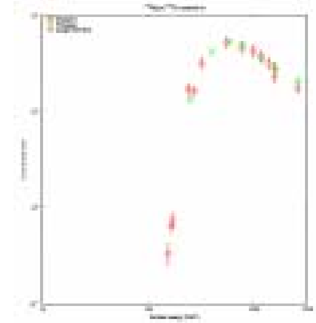


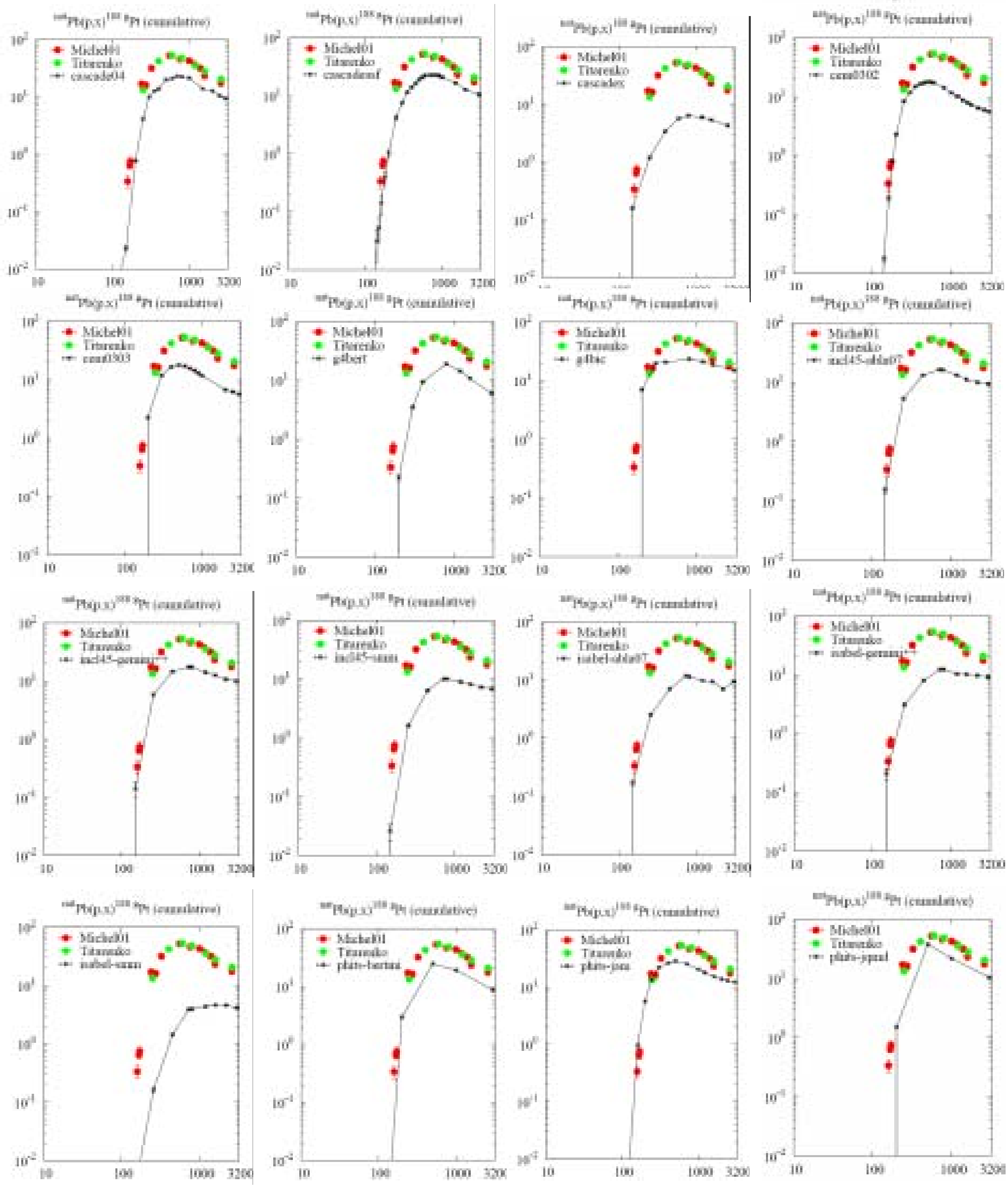
# Hf-175 from lead



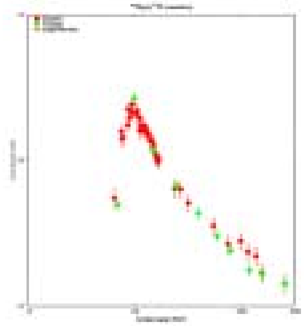


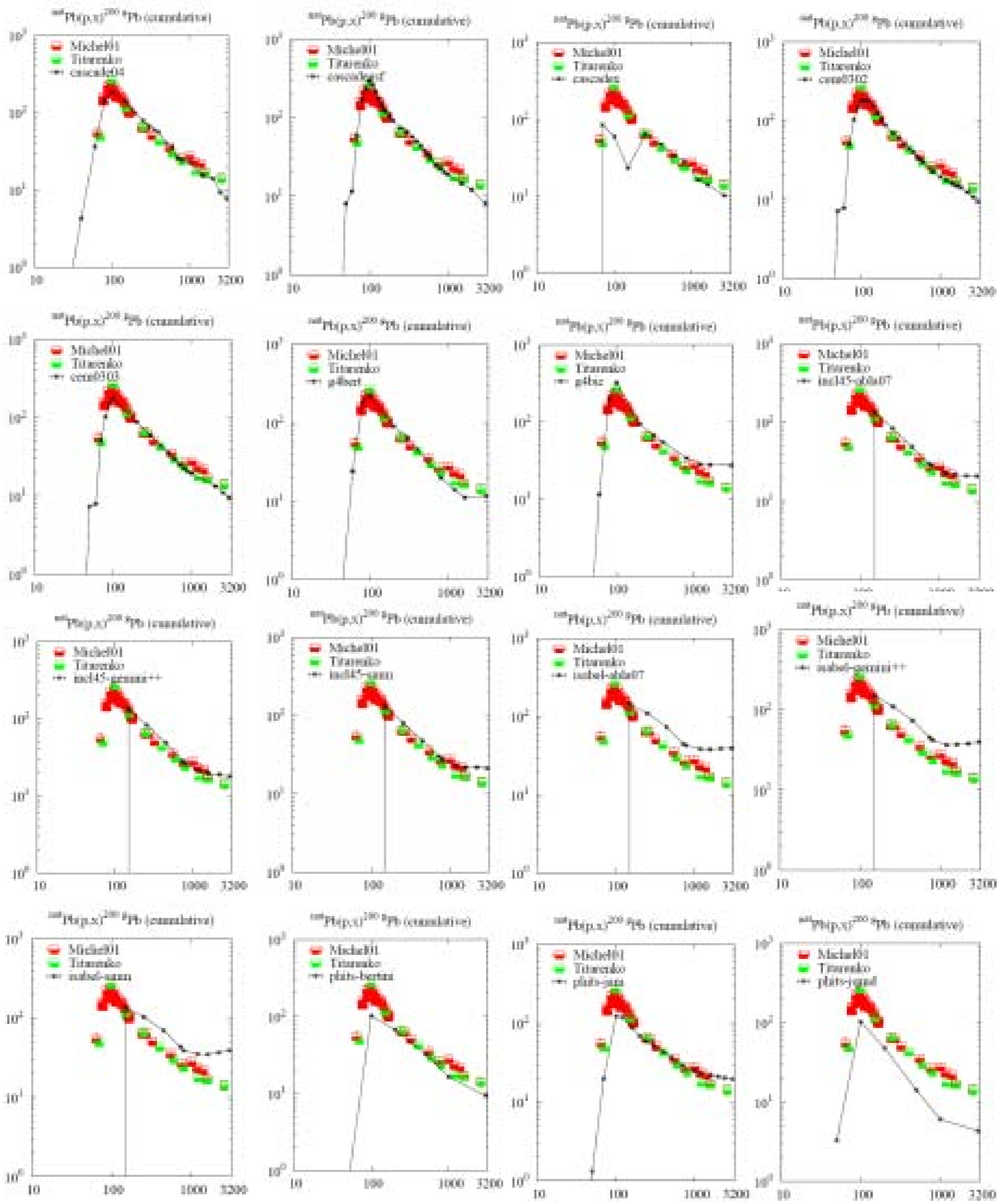
# Pt-188 from lead



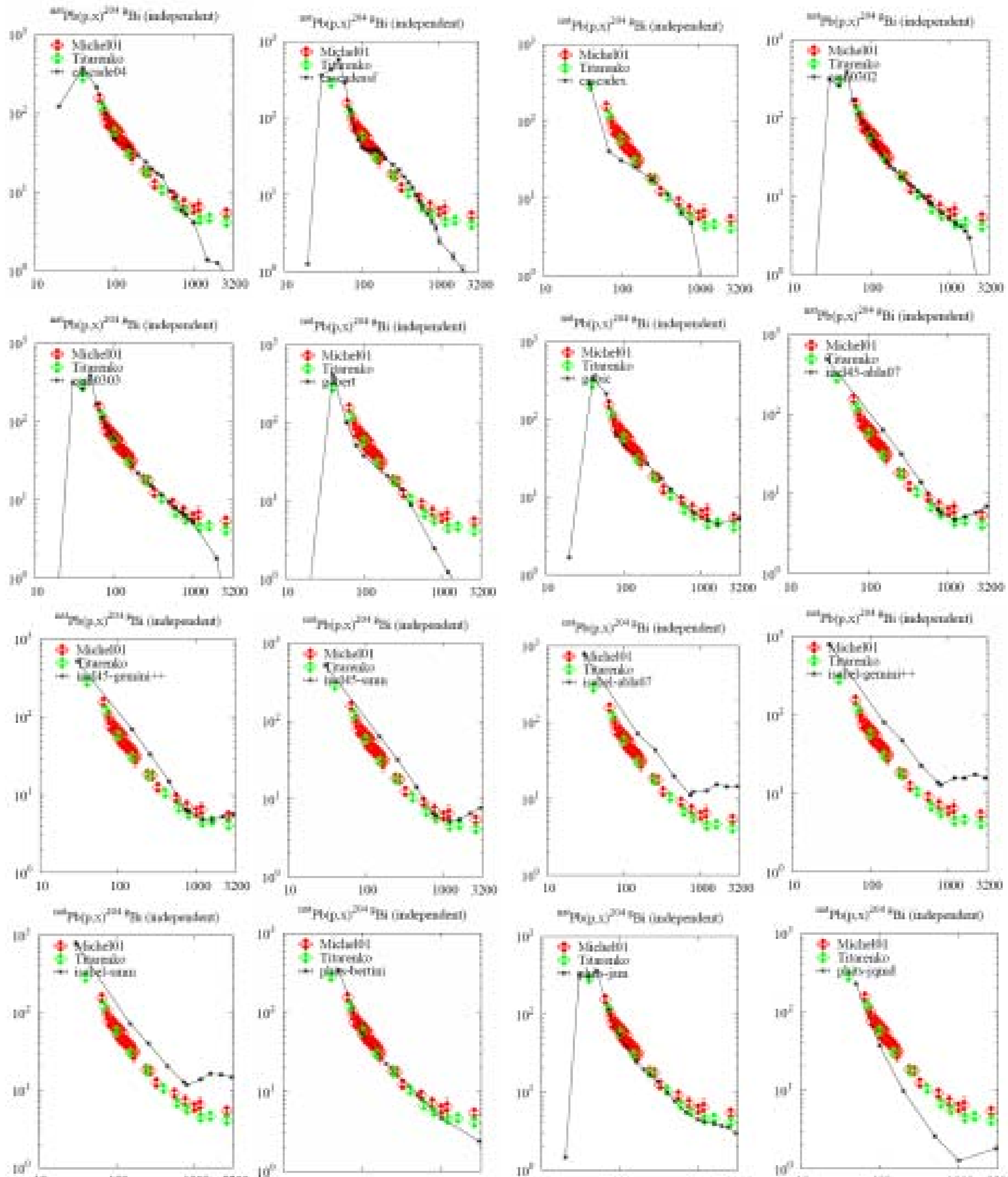


# Pb-200 from lead



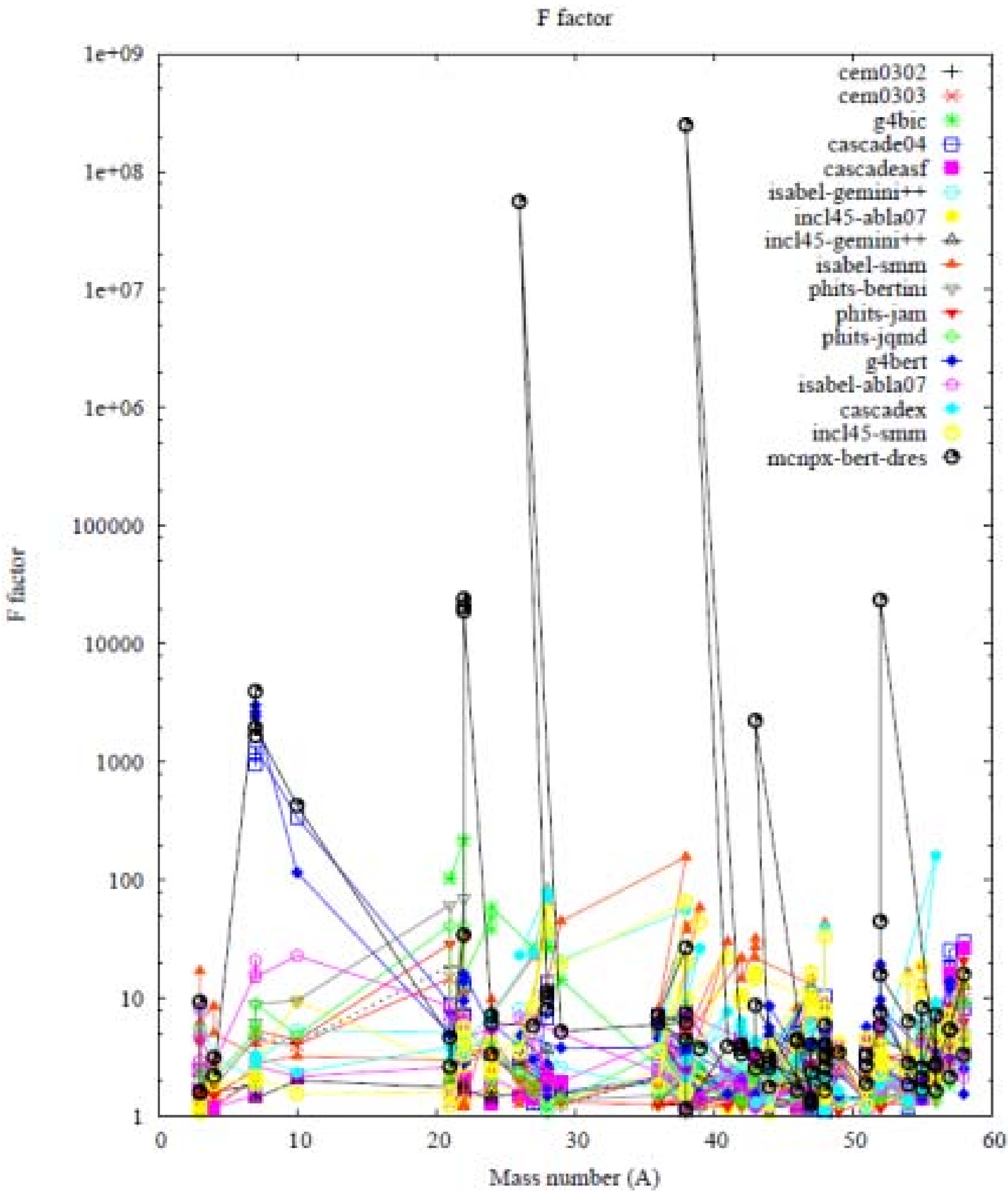


# Bi-204 from lead

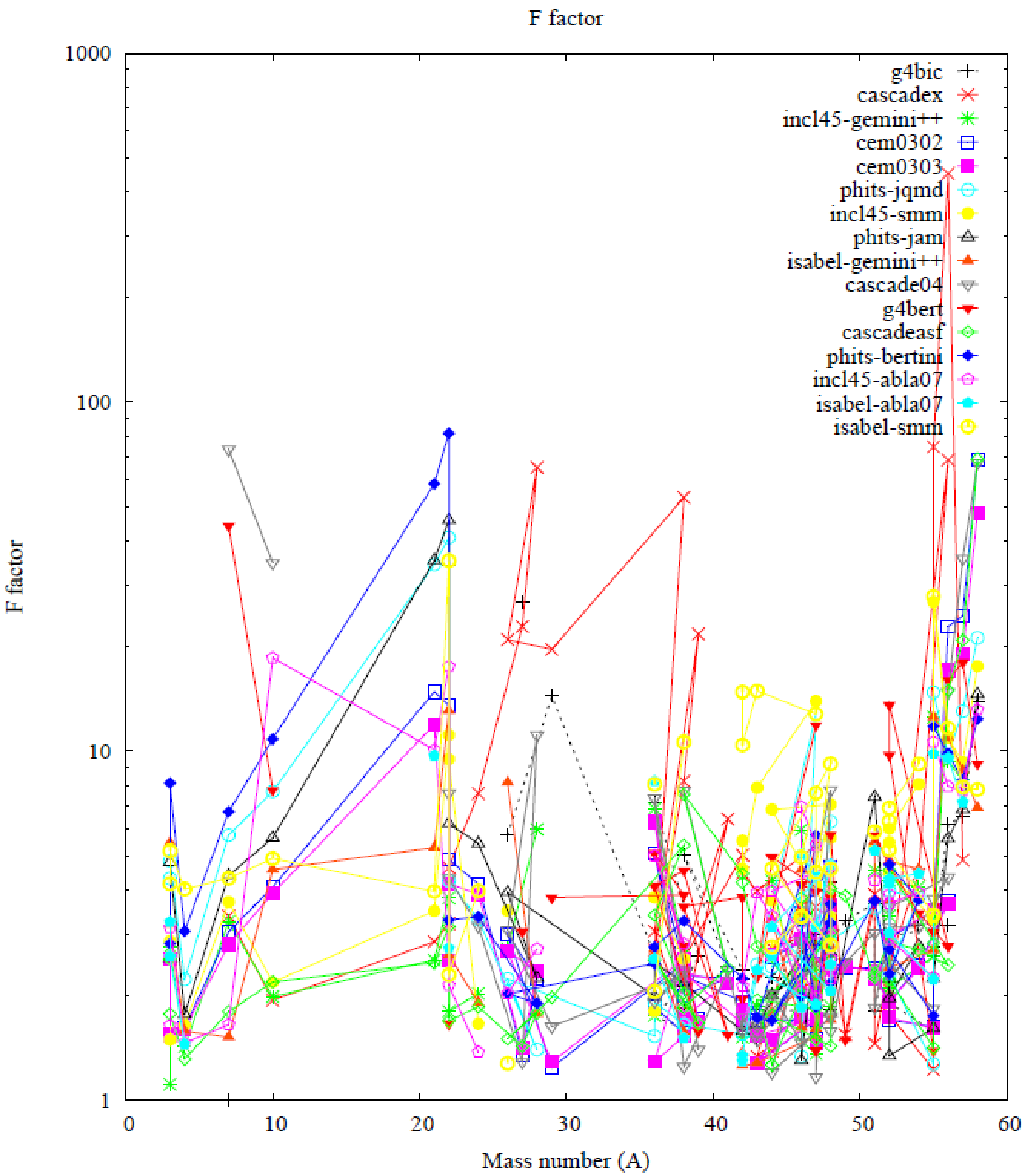


# Statistical Factors

# Mean deviation factor F exctation functions iron

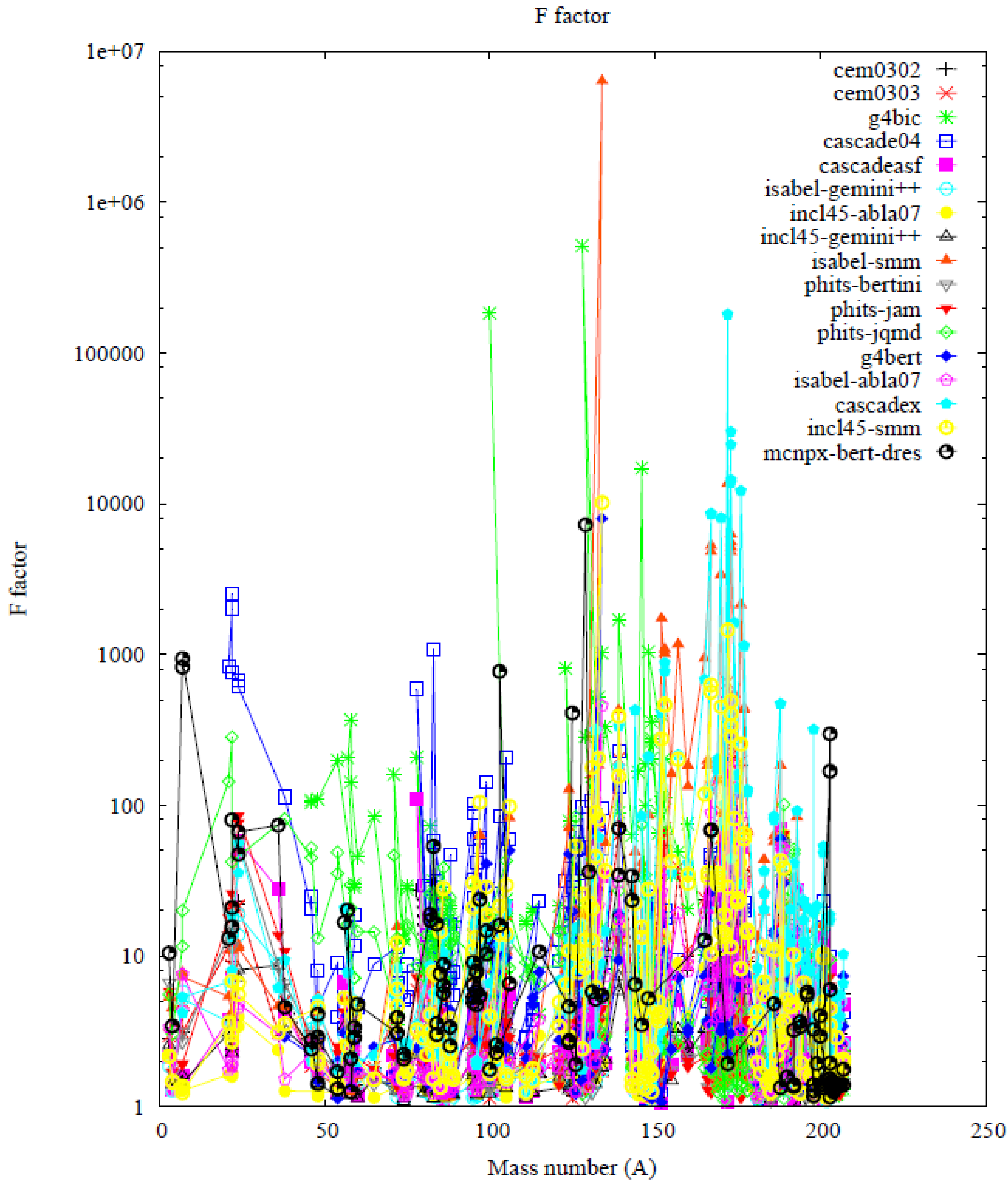


# Mean deviation factor F exctation functions iron

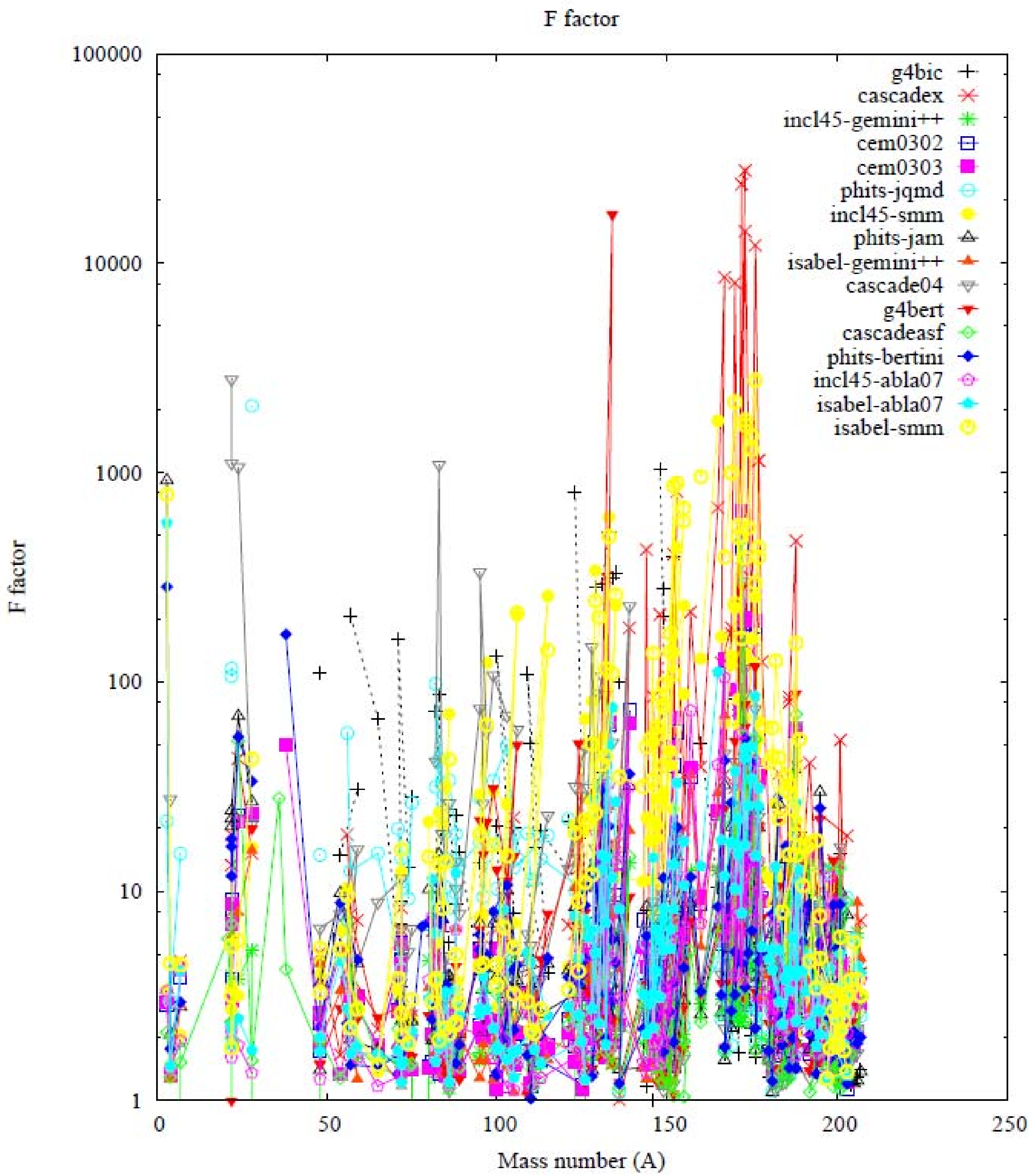



**without mcnpx-bert**

# Mean deviation factor F exctation functions lead



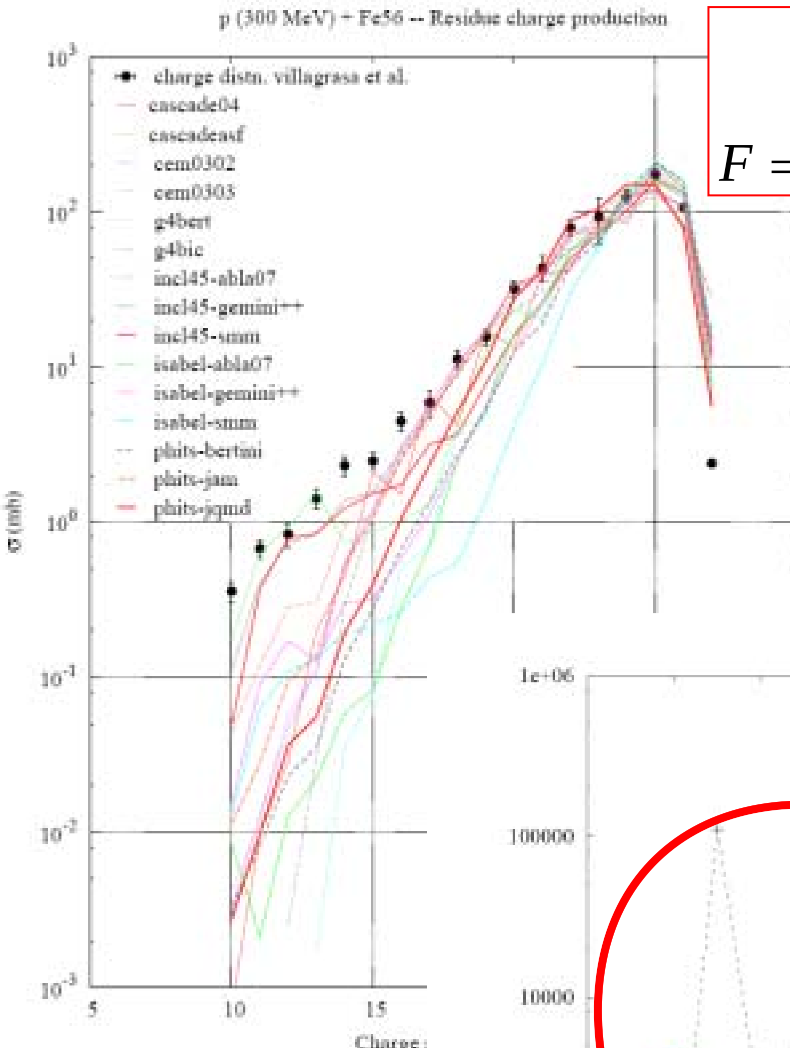
# Mean deviation factor F exctation functions lead

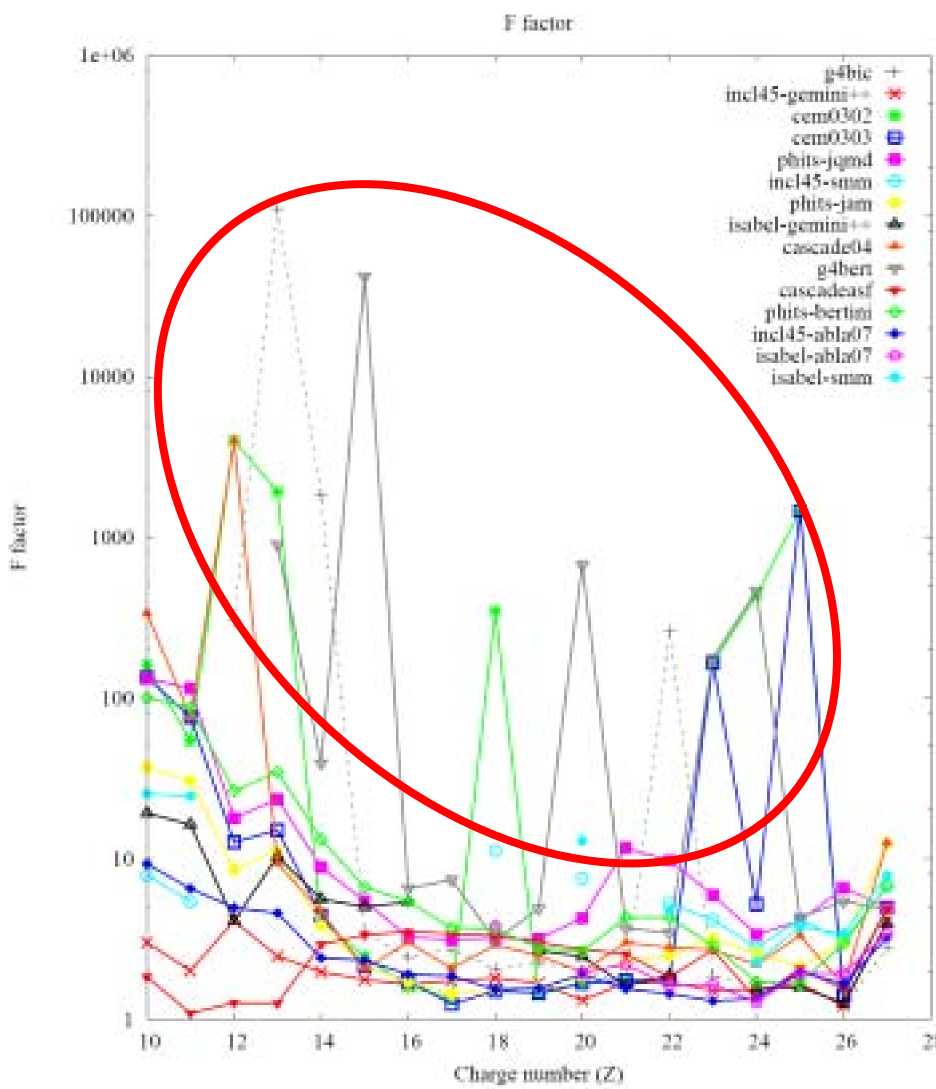



**without mcnpx-bert**

# F-Factor

$$F = 10^{\left(\frac{1}{N}\sum_{i=1}^{N}\left[\log\left(\sigma_i^{\exp}\right)-\log\left(\sigma_i^{calc}\right)\right]^2\right)^{1/2}}$$



# The Concept of Intrinsic Discrepancy

I propose to replace

the mean deviation factor

by the intrinsic discrepancy between

the experimental and theoretical PDFs.

**PDF = probability density function**

# The Concept of Intrinsic Discrepancy

**The intrinsic discrepancy $\delta\{p_1,p_2\}$ is a very general measure of the divergence between two distributions of the random vector $\mathbf{x}$ described by their density functions $p_1$ and $p_2$:**

$$\delta\{p_1, p_2\} = \min\left\{\int p_1(x)\ln\frac{p_1(x)}{p_2(x)}\mathrm{d}x, \int p_2(x)\ln\frac{p_2(x)}{p_1(x)}\mathrm{d}x\right\}$$

**J.M. Bernardo, Bayesian Statistics, in: *Probability and Statistics* (R. Viertl, ed.) *Encyclopedia of Life Support Systems* (EOLSS). Oxford, UK: UNESCO, 2003.**

# Cross Sections:
## Excitation functions as PDFs

Let $\sigma(E)$ be integrable

in a given energy interval

$[E_{\min}, E_{\max}]$, then :

$$\sigma'(E) = \frac{\sigma(E)}{\int_{E_{\min}}^{E_{\max}} \sigma(E)\,\mathrm{d}E} = f(E)$$

**is a PDF for a reaction to occur as a function of energy $E$.**

# The Concept of Intrinsic Discrepancy applied to residual nuclide production

$$\sigma'_{\mathrm{exp}}(x) = \frac{\sigma_{\mathrm{exp}}(x)}{\int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} \sigma_{\mathrm{exp}}(x)\,\mathrm{d}x} \qquad \sigma'_{\mathrm{calc}}(x) = \frac{\sigma_{\mathrm{calc}}(x)}{\int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} \sigma_{\mathrm{calc}}(x)\,\mathrm{d}x}$$

$$\delta\{\sigma'_{\mathrm{exp}}(x), \sigma'_{\mathrm{calc}}(x)\} =$$

$$\min\left\{\int \sigma'_{\mathrm{exp}}(x) \ln\frac{\sigma'_{\mathrm{exp}}(x)}{\sigma'_{\mathrm{calc}}(x)}\mathrm{d}x, \int \sigma'_{\mathrm{calc}}(x) \ln\frac{\sigma'_{\mathrm{calc}}(x)}{\sigma'_{\mathrm{exp}}(x)}\mathrm{d}x\right\}$$

with $x = E, A, Z, \ldots$

# The Concept of Intrinsic Discrepancy applied to excitation functions

$$\sigma'_{\mathrm{exp}}(E) = \frac{\sigma_{\mathrm{exp}}(E)}{\int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} \sigma_{\mathrm{exp}}(E)\,\mathrm{d}x} \qquad \sigma'_{\mathrm{calc}}(E) = \frac{\sigma_{\mathrm{calc}}(E)}{\int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} \sigma_{\mathrm{calc}}(E)\,\mathrm{d}x}$$

$$\delta\{\sigma'_{\mathrm{exp}}(E), \sigma'_{\mathrm{calc}}(E)\} = \min\left\{\int \sigma'_{\mathrm{exp}}(E) \ln \frac{\sigma'_{\mathrm{exp}}(E)}{\sigma'_{\mathrm{calc}}(E)}\,\mathrm{d}x, \int \sigma'_{\mathrm{calc}}(E) \ln \frac{\sigma'_{\mathrm{calc}}(E)}{\sigma'_{\mathrm{exp}}(E)}\,\mathrm{d}x\right\}$$

$$\delta\{\sigma'_{\mathrm{exp}}(E), \sigma'_{\mathrm{calc}}(E)\} = \min\left\{ \begin{aligned} &\left(\int \sigma'_{\mathrm{exp}}(E) \ln \frac{\sigma_{\mathrm{exp}}(E)}{\sigma_{\mathrm{calc}}(E)}\,\mathrm{d}E + \ln \frac{\int \sigma_{\mathrm{calc}}(E)\mathrm{d}E}{\int \sigma_{\mathrm{exp}}(E)\mathrm{d}E}\right), \\ &\left(\int \sigma'_{\mathrm{calc}}(E) \ln \frac{\sigma_{\mathrm{calc}}(E)}{\sigma_{\mathrm{exp}}(E)}\,\mathrm{d}E + \ln \frac{\int \sigma_{\mathrm{exp}}(E)\mathrm{d}E}{\int \sigma_{\mathrm{calc}}(E)\mathrm{d}E}\right) \end{aligned} \right\}$$

# Intrinsic Discrepancy: Characteristics

- It may be shown that the intrinsic divergence is symmetric, non-negative, and it is zero if, and only if, $p_1(x) = p_2(x)$ almost everywhere.

- The intrinsic discrepancy is invariant under one-to-one transformations of $x$.

- Besides, it is additive: if $x = \{x_1, \ldots, x_n\}$ and

$$p_i(\mathbf{x}) = \prod_{j=1}^{n} q_i(x_j)$$

, then $\delta\{p_1, p_2\} = n\,\delta\{q_1, q_2\}$.

- Last, but not least, it is defined even if the support of one of the densities is strictly contained in the support of the other.

# Intrinsic Discrepancy: Characteristics

**The intrinsic discrepancy serves to define a useful type of convergence; a sequence of PDFs**

$\{p_i(\mathbf{x})\}_{i=1}^{\infty}$ ***converges intrinsically***

**to a PDF $p(x)$ if (and only if)**

$$\lim_{i\to\infty} \delta(p_i, p) = 0$$

***i.e.*, if (and only if) the sequence of the corresponding intrinsic discrepancies converges to zero.**

# Benchmark of Spallation Models

## Results of a global analysis: Residues

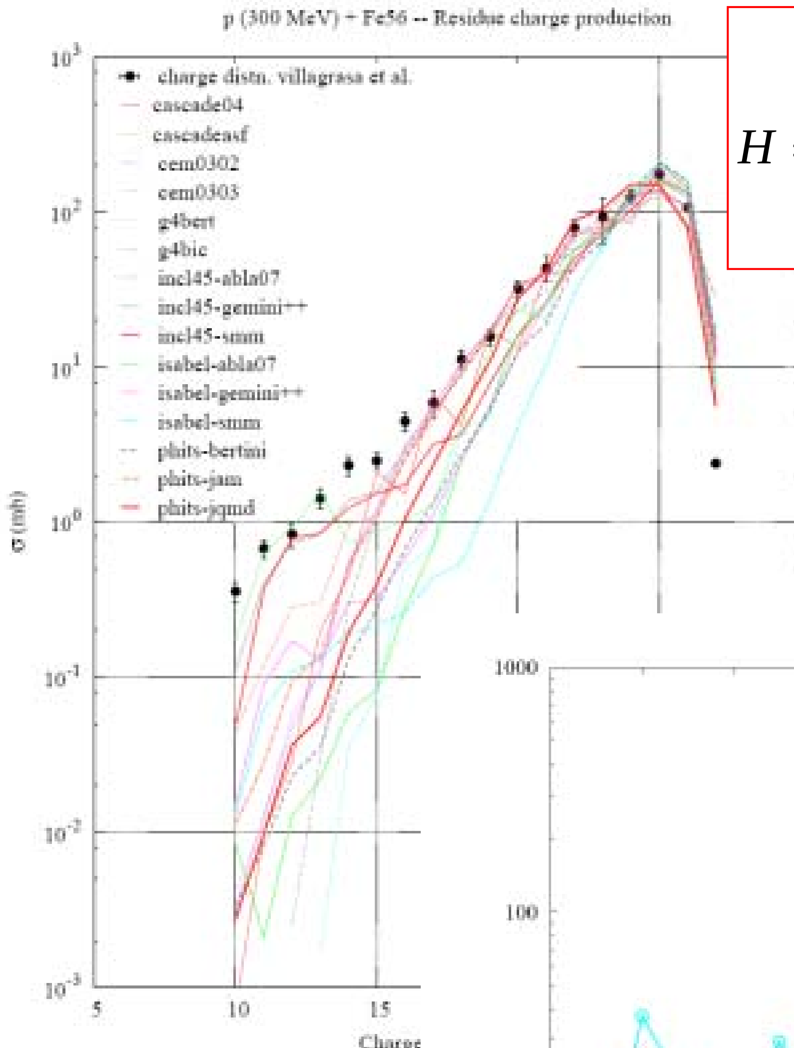
# Conclusion

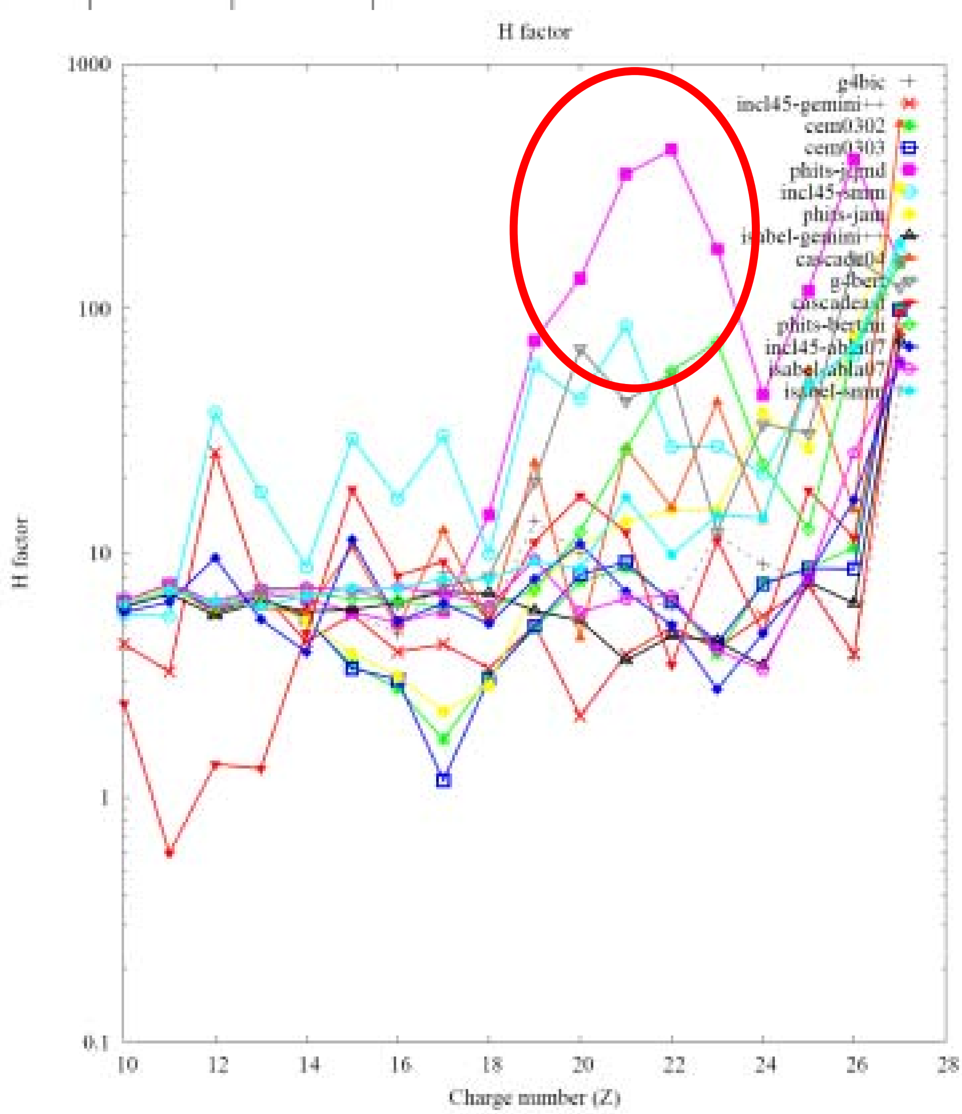
**There is hope,**
**but there is still room for improvements.**

# Deviation Factors

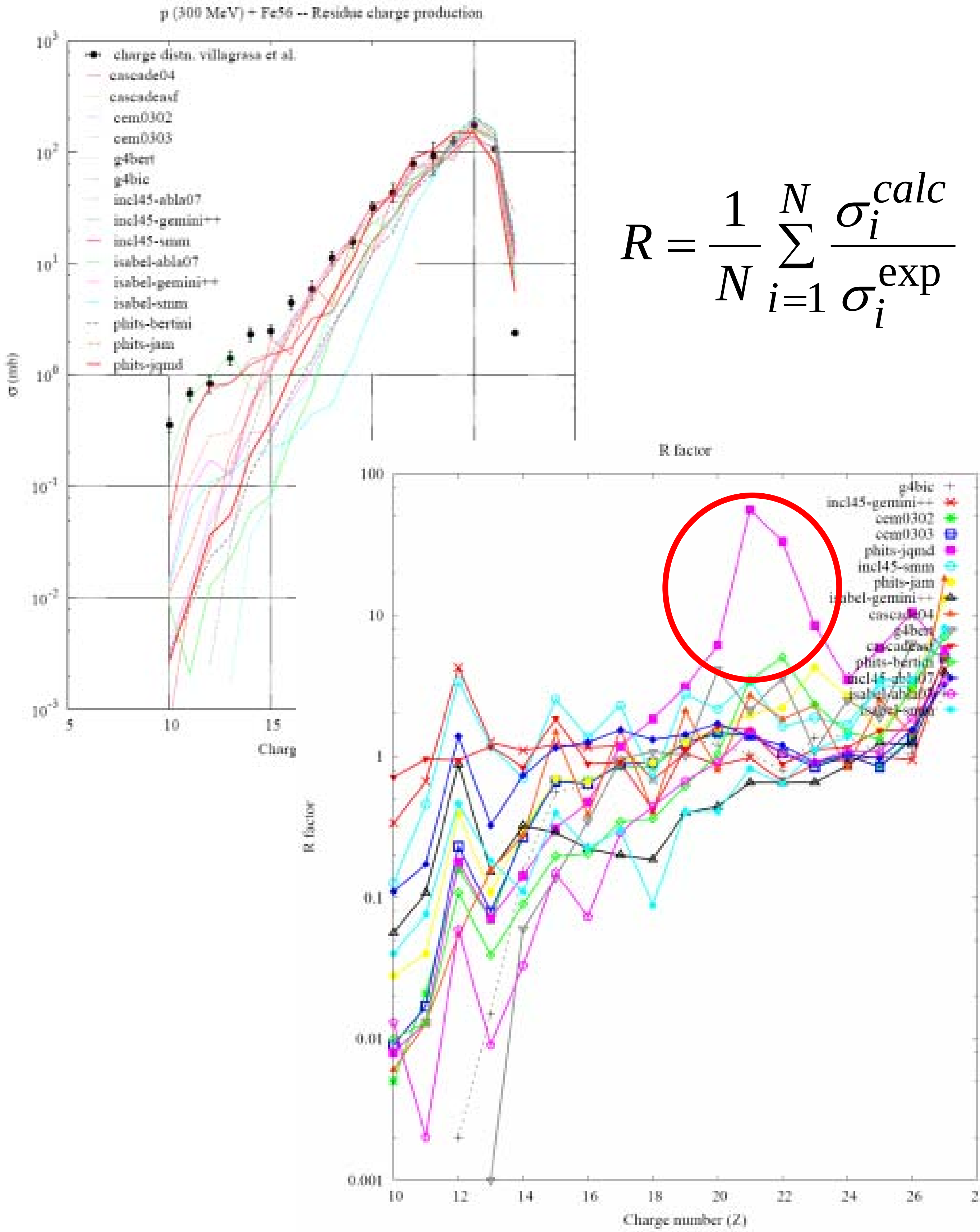
# H-Factor

$$H = \left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\sigma_i^{\exp} - \sigma_i^{calc}}{\Delta\sigma_i^{\exp}} \right)^2 \right)^{1/2}$$



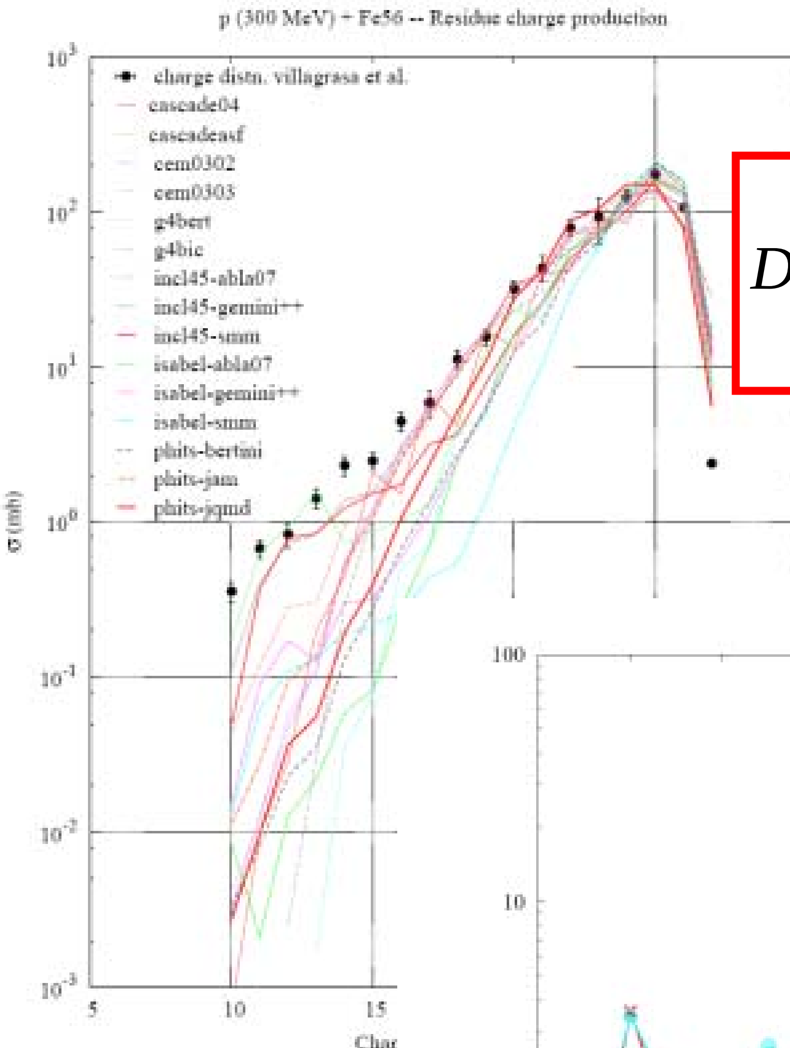



# R-Factor

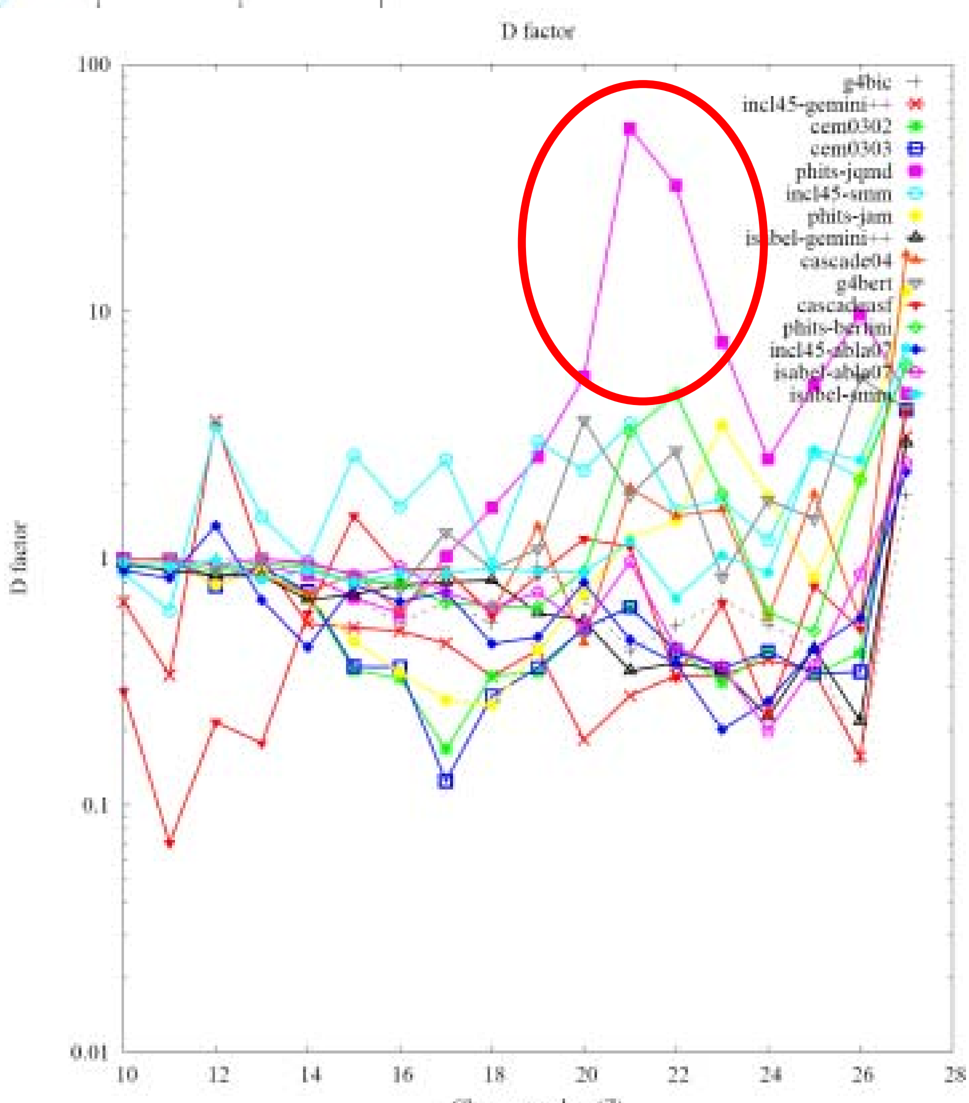



$$R = \frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_i^{calc}}{\sigma_i^{\exp}}$$

# D-Factor




$$D = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\sigma_i^{\exp} - \sigma_i^{calc}}{\sigma_i^{\exp}}\right|$$



# L-Factor

$$L = \left[ \sum_{i=1}^{N} \left( \frac{\sigma_i^{calc}}{\Delta \sigma_i^{\exp}} \right)^2 \left( \frac{\sigma_i^{calc} - \sigma_i^{\exp}}{\sigma_i^{calc}} \right)^2 \Bigg/ \sum_{i=1}^{N} \left( \frac{\sigma_i^{calc}}{\Delta \sigma_i^{\exp}} \right)^2 \right]^{1/2}$$